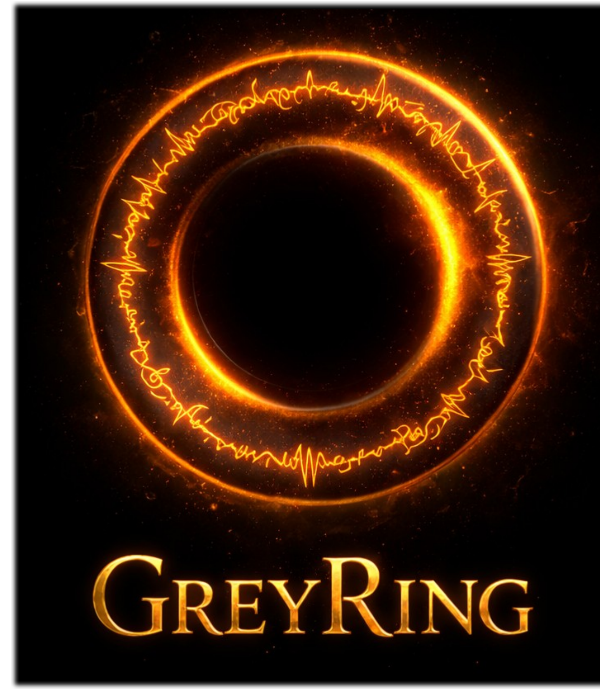
GREYRING: Novel ringdown tests of general relativity with black hole greybody factors

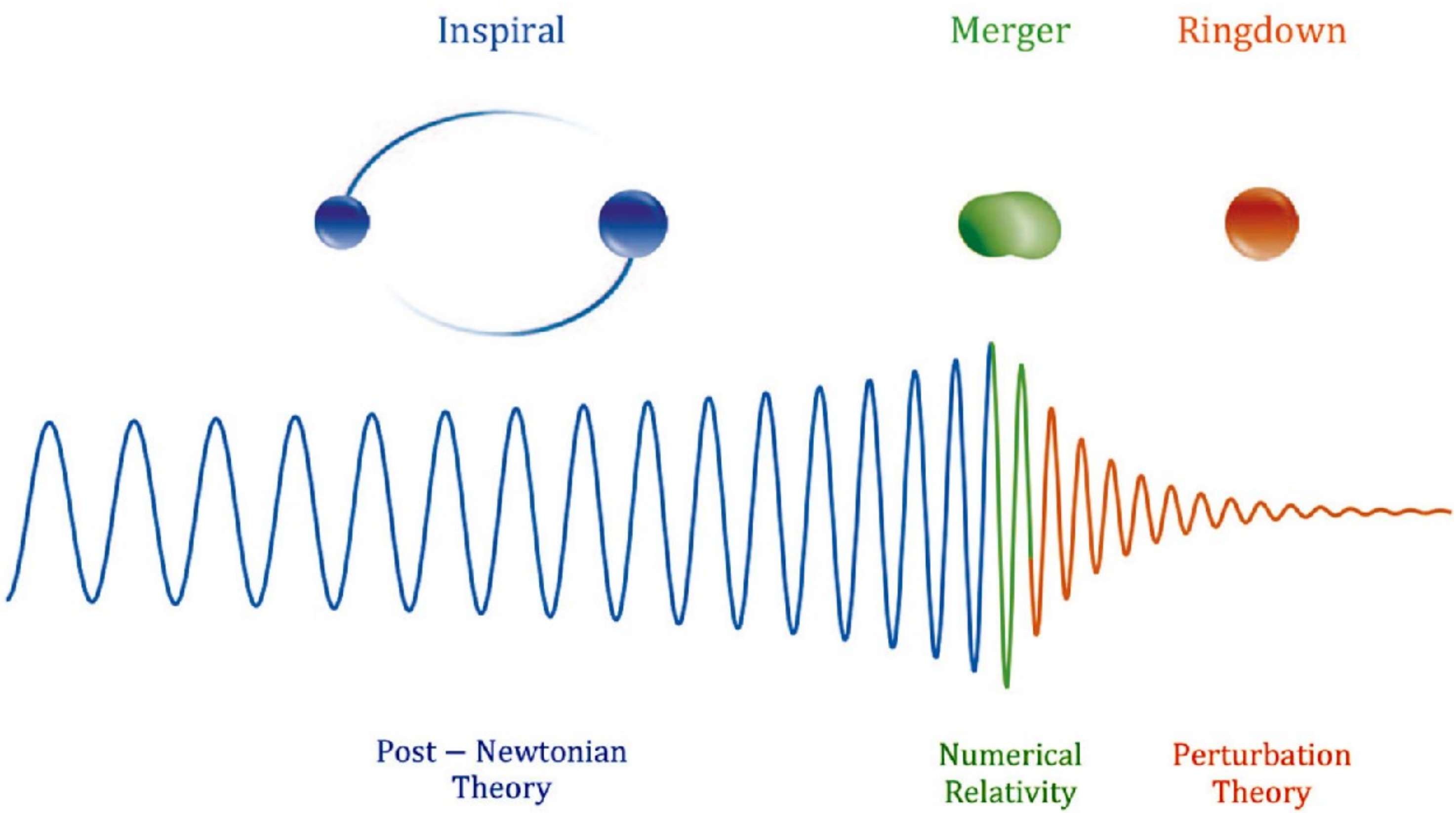


2406.01692; 2501.16433; 2511.08692; 2512.15877; 2603.20490; 2604.11895
with **R.F. Rosato**, K. Destounis, S. Biswas, S. Chakraborty, E. Berti, S. Yi, M. De Amicis, F. Crescimbeni

ET-NOW FIS2

Paolo Pani
Sapienza University of Rome & INFN Roma1

Outline



- Black hole (BH) ringdown & subtleties with quasinormal modes (QNMs)
- Greybody factors as a novel ringdown probe
- Greybody factors & novel tests of gravity
- Bonus: Black Hole Amplitudescopy

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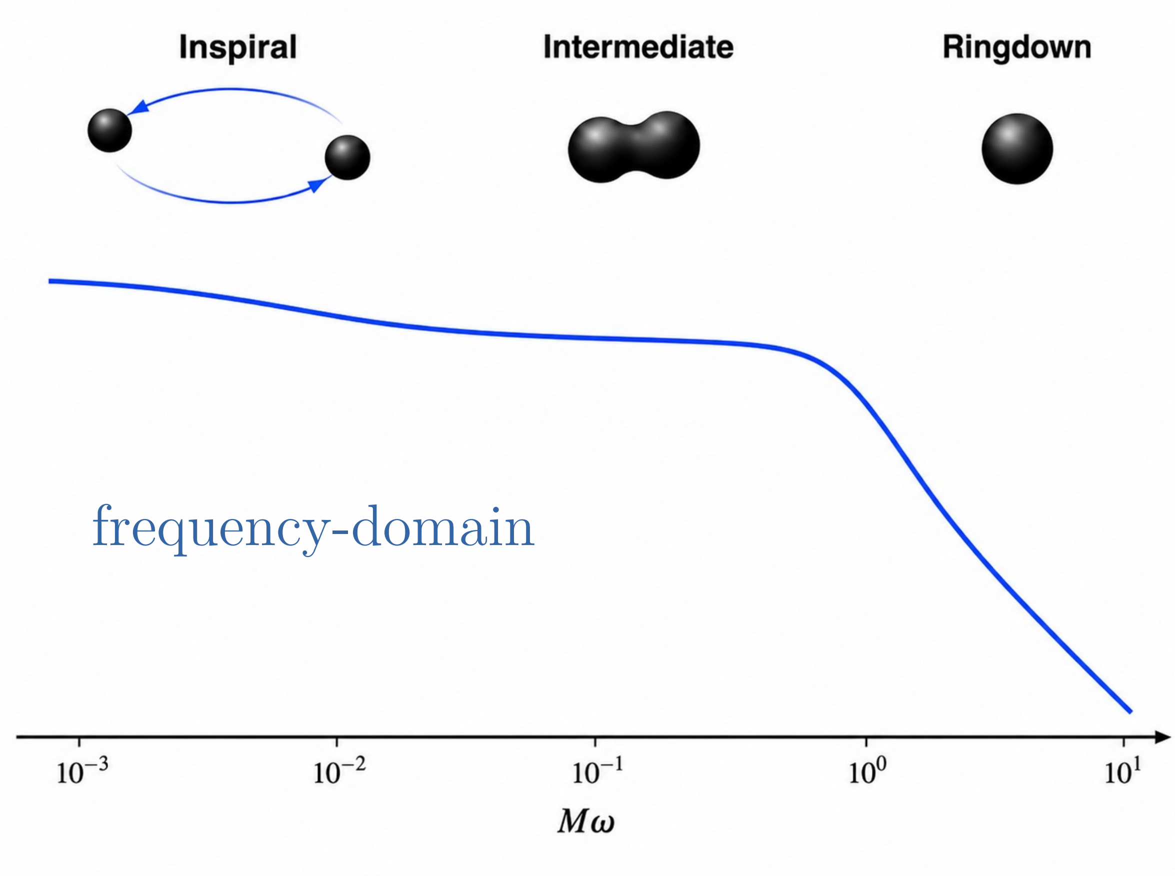

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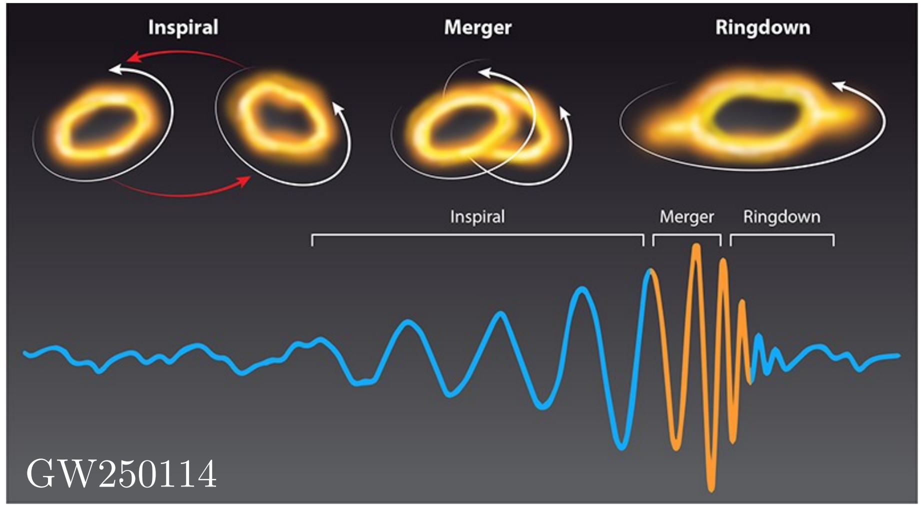
Ringdown is simple... or not?



TOPICAL REVIEW

CQG 2009

Quasinormal modes of black holes and black branes

Emanuele Berti[1,2], Vitor Cardoso[1,3], Andrei O. Starinets[4]

- ~15 yr ago: ringdown is simple and clean...

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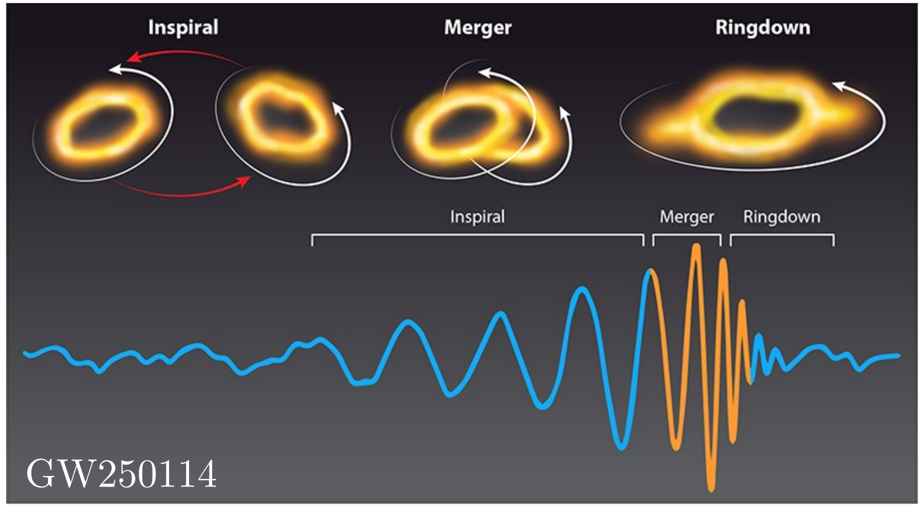

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  - Overtones, starting time, overfitting, mode mixing, echoes, nonlinearities, spectral instabilities, environmental effects, redshift/horizon, direct waves...
  - Accurate ringdown modelling requires a lot of ingredients

Ringdown is simple... or not?

Black hole spectroscopy: from theory to experiment

Emanuele Berti[1], Vitor Cardoso[2,3], Gregorio Carullo[2,4], Jahed Abedi[5], Niayesh Afshordi[6,7,8], Simone Albanesi[9,10], Vishal Baibhav[11], Swetha Bhagwat[4], José Luis Blázquez-Salcedo[12], Béatrice Bonga[13], Bruno Bucciotti[14,15], Giada Caneva Santoro[16], Pablo A. Cano[17], Collin Capano[18,19], Mark Ho-Yeuk Cheung[1], Cecilia Chirenti[20,21,22,23], Gregory B. Cook[24], Adrian Ka-Wai Chung[25], Marina De Amicis[2], Kyriakos Destounis[3], Oscar J. C. Dias[26], Walter Del Pozzo[27,15], Francisco Duque[28], Will M. Farr[29,30], Eliot Finch[31], Nicola Franchini[32,3], Kwinten Fransen[33], Vasco Gennari[34], Stephen R. Green[35], Scott A. Hughes[36], Maximiliano Isi[30], Xisco Jimenez Forteza[37], Gaurav Khanna[38,19], Fech Scen Khoo[12,39], Masashi Kimura[40,41], Badri Krishnan[42,13], Adrien Kuntz[43,44,45,3], Macarena Lagos[46], Rico K. L. Lo[2], Lionel London[47], Sizheng Ma[8], Simon Maenaut[48,49], Lorena Magaña Zertuche[2], Elisa Maggio[28], Andrea Maselli[50,51], Keefe Mitman[52], Hayato Motohashi[53], Naritaka Oshita[54,55,56], Costantino Pacilio[57,58], Paolo Pani[59], Rodrigo Panosso Macedo[2], Chantal Pitte[60,43,44,45], Lorenzo Pompili[28], Jaime Redondo-Yuste[2], Maurício Richartz[23], Antonio Riotto[61], Jorge E. Santos[62], Bangalore Sathyaprakash[63,64,65], Laura Sberna[35], Hector O. Silva[25,28], Leo C. Stein[66], Alexandre Toubiana[57,58], Sebastian H. Völkel[28], Julian Westerweck[4], Huan Yang[67], Sophia Yi[1], Nicolas Yunes[25] and Hengrui Zhu[68]

Class. Quantum Grav. 43 123001 (2026) (today)

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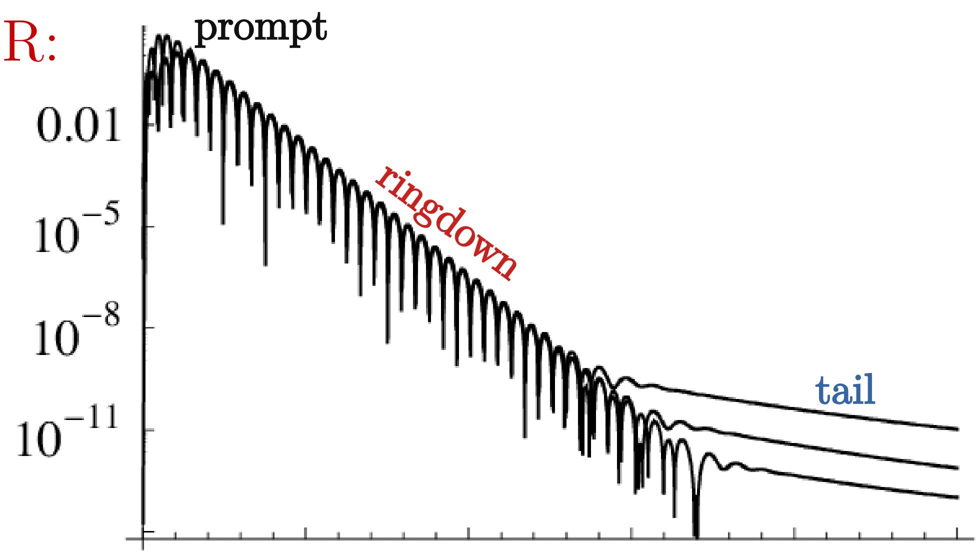
BH ringdown in a nutshell

- Perturbations on given background, in GR:

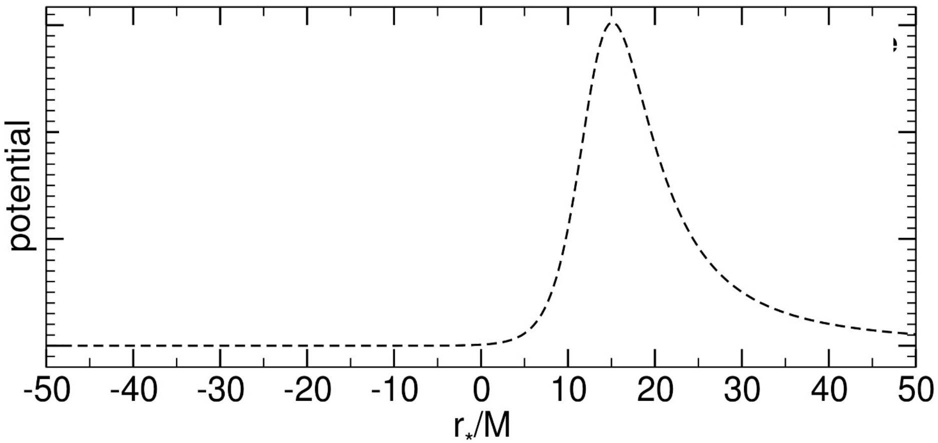
$$R^{\rm bkg}_{\mu\nu} = 0 \quad \text{Kerr BH}$$

$$\Box_{\rm bkg} h_{\mu\nu} = 0 \quad \text{Linear perturbations + boundary conditions}$$



$$\frac{\partial^2\Psi}{\partial t^2} - \frac{\partial^2\Psi}{\partial r_*^2} + V_{slm}(r_*)\Psi = S$$

time domain



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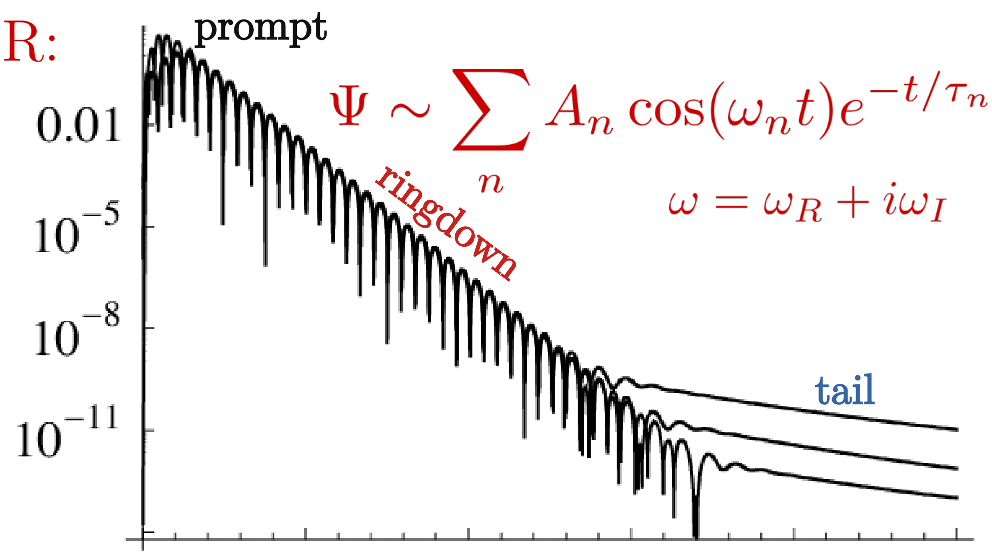
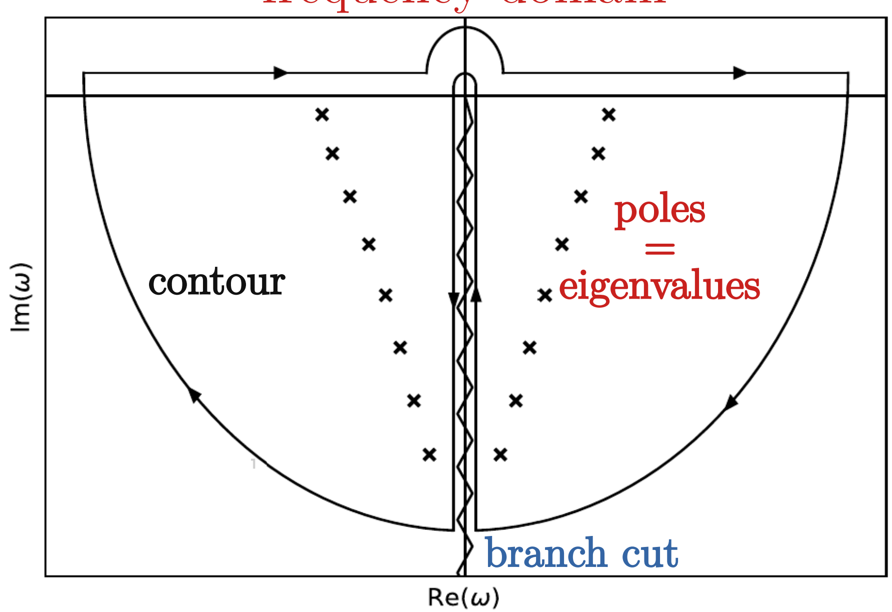


$$\frac{\partial^2 \tilde{\Psi}}{\partial r_*^2} + (\omega^2 - V_{slm}(r_*))\tilde{\Psi} = \tilde{S}$$

frequency domain



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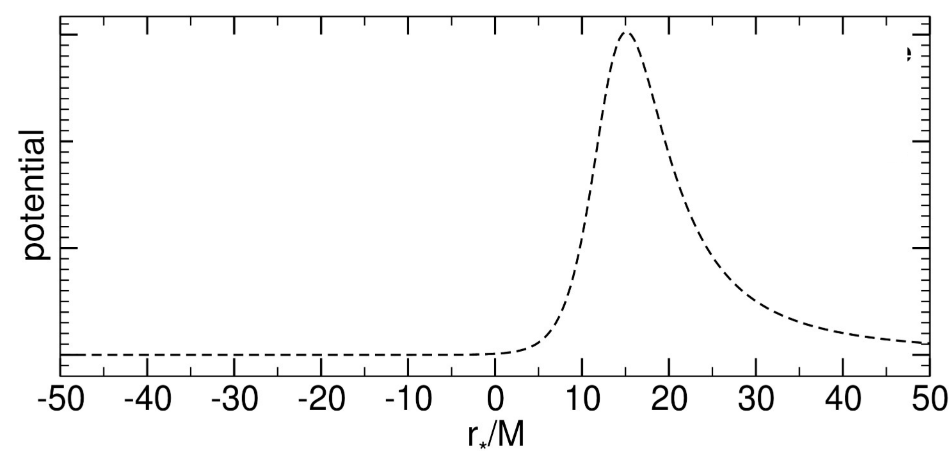
time domain



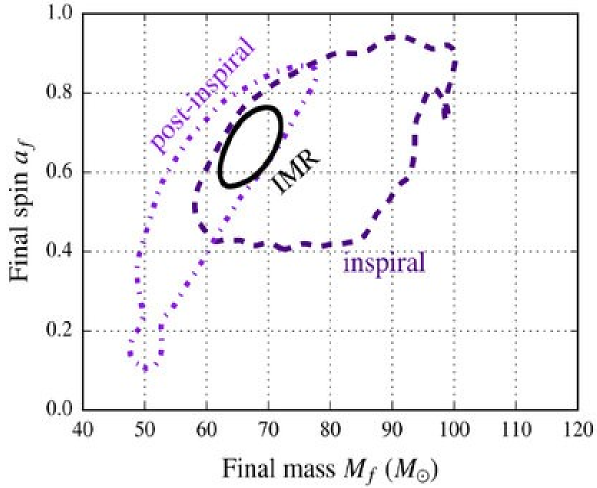
Ringdown tests of gravity

Beyond GR/Kerr: Shift of QNMs (bkg geometry + dynamics + boundary conditions):

$$\omega_{lmn} = \omega_{lmn}^{\rm Kerr}(M,\chi) + \delta\omega_{lmn}(M,\chi,\ell_{\rm new})$$

LVK:

- Dominant QNM measured in various events
- Overtone detected in GW250114 at O(10)% [LVK PRL 2026]



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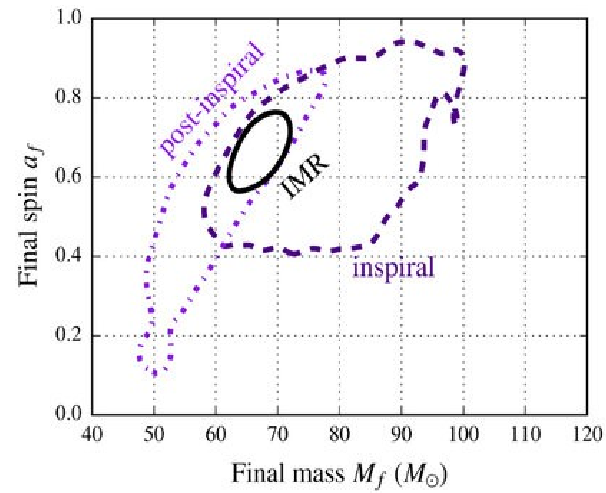
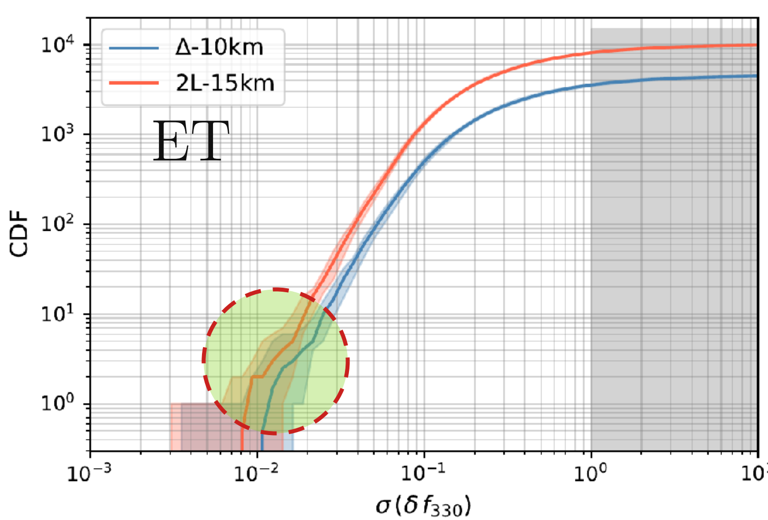
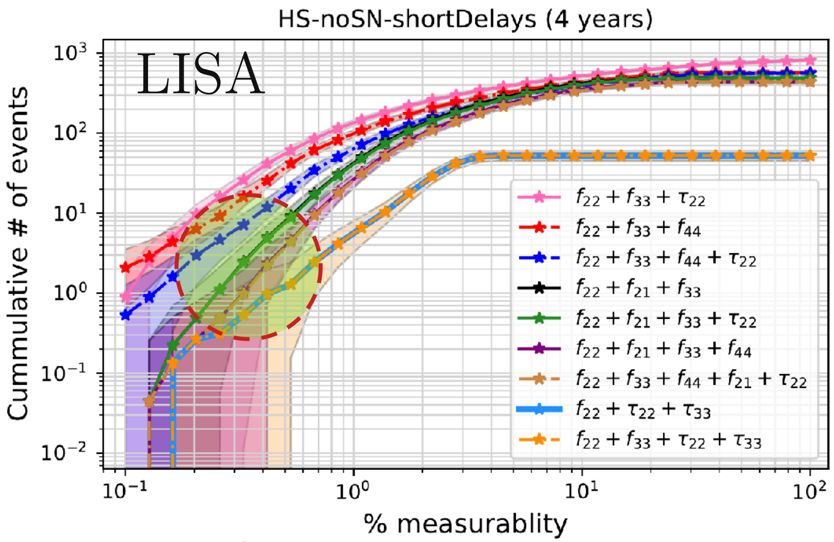


ET/LISA:

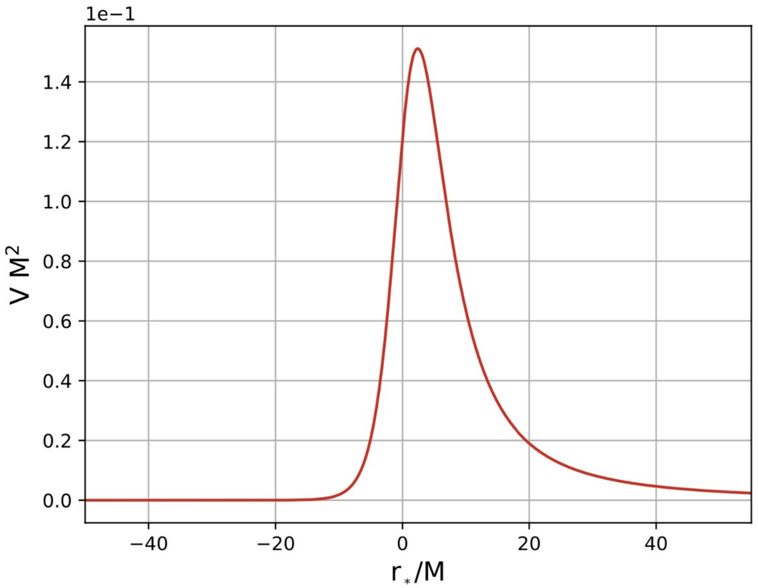
- 3+ QNMs @ (sub)percent level for O(1-100) events/yr

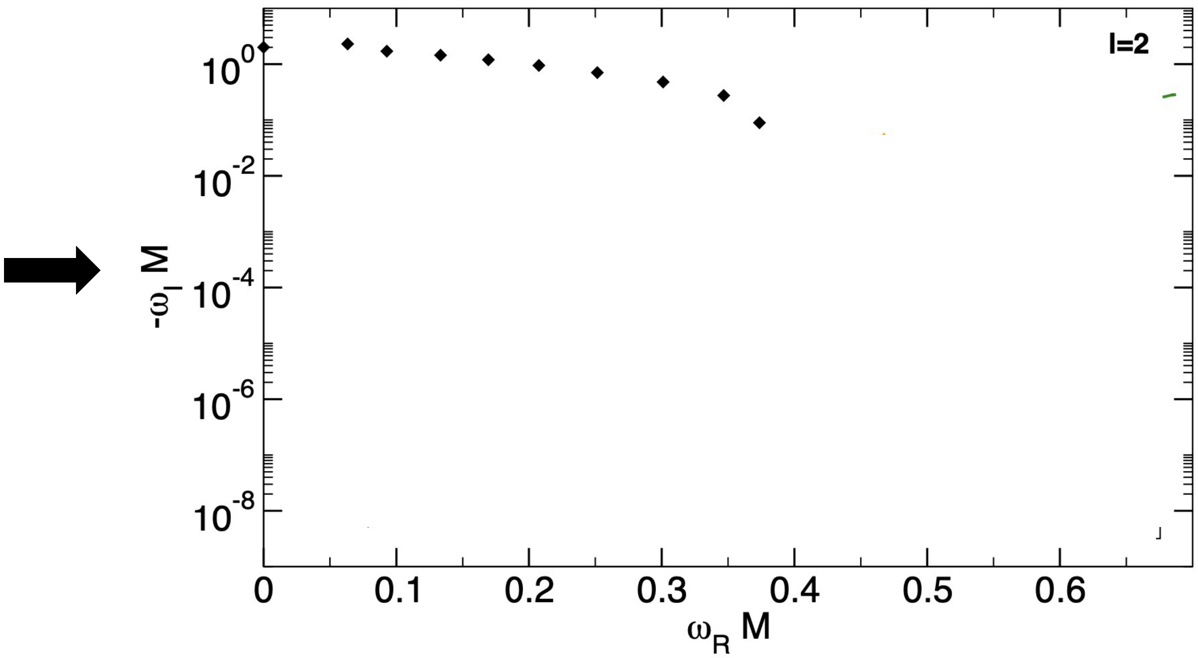




[Bhagwat+ PRD 2022, Bhagwat+ PRD 2023]

QNM spectral instability





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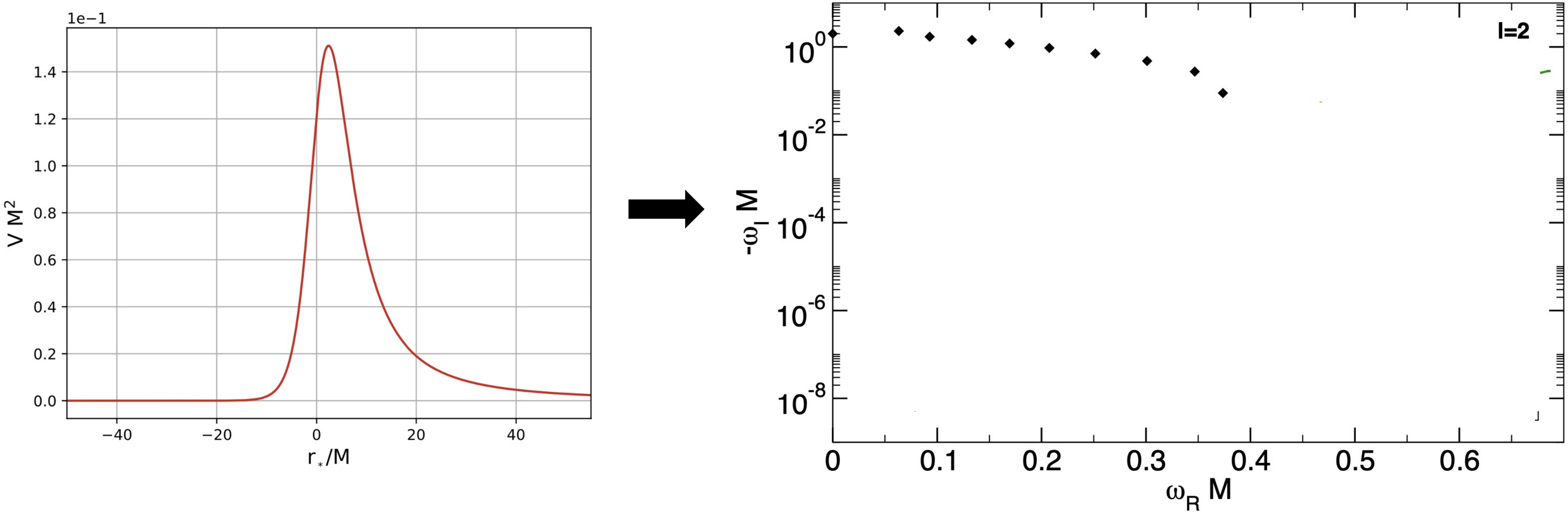

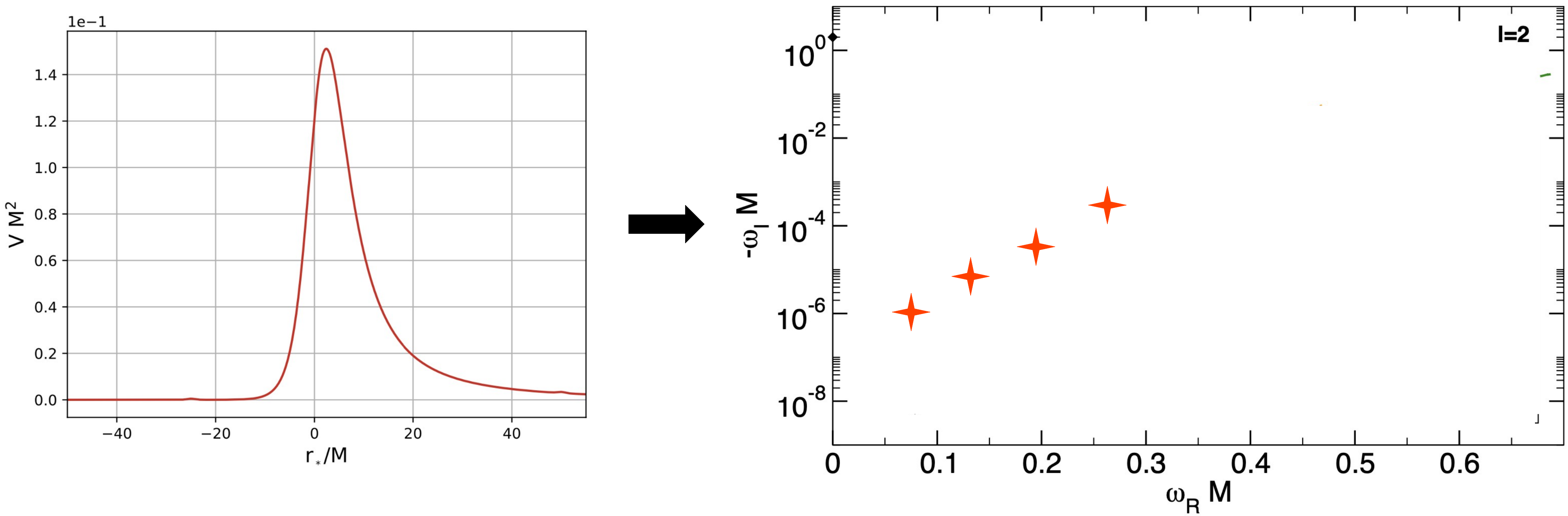

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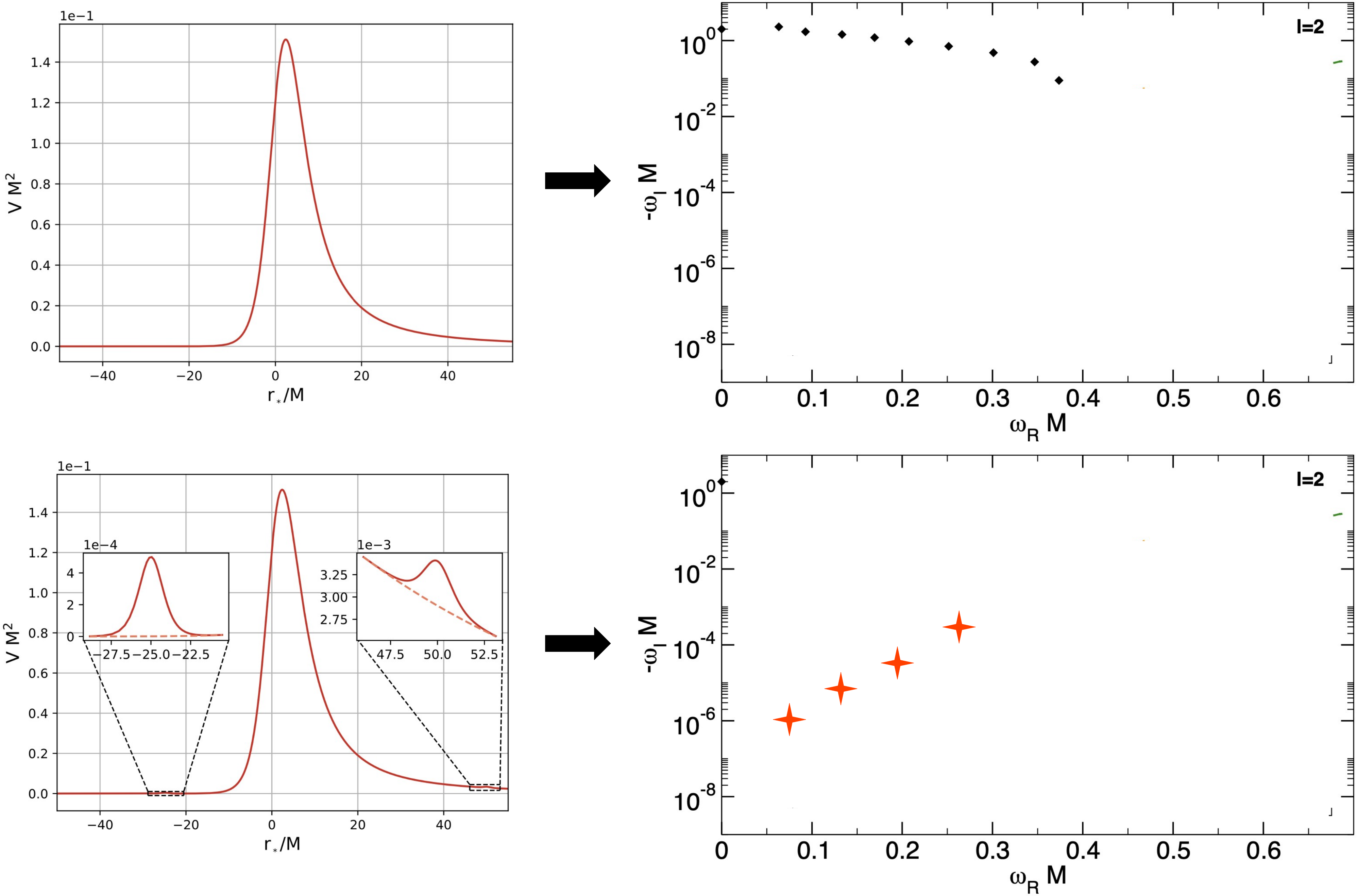
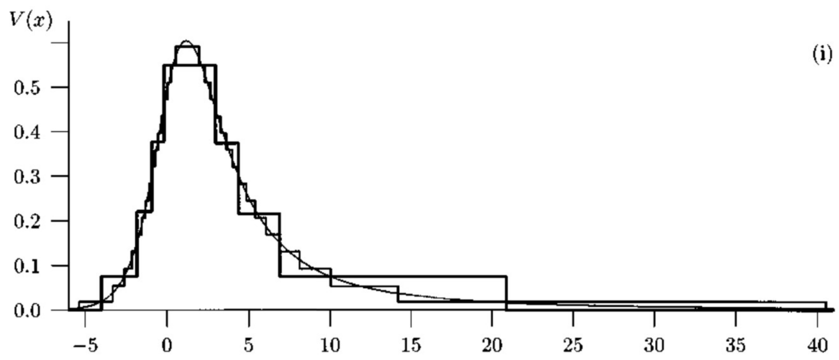
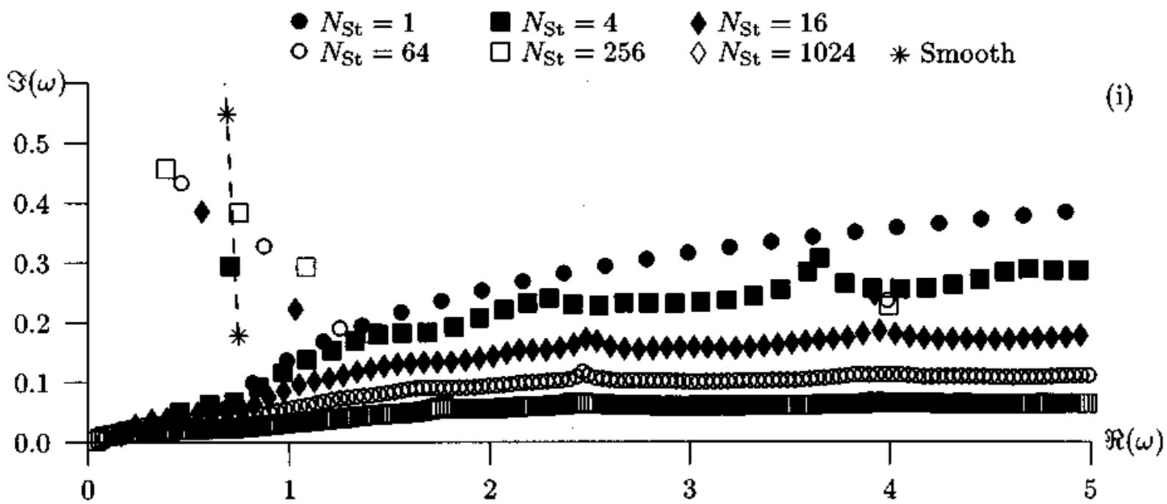


QNM spectral instability #2

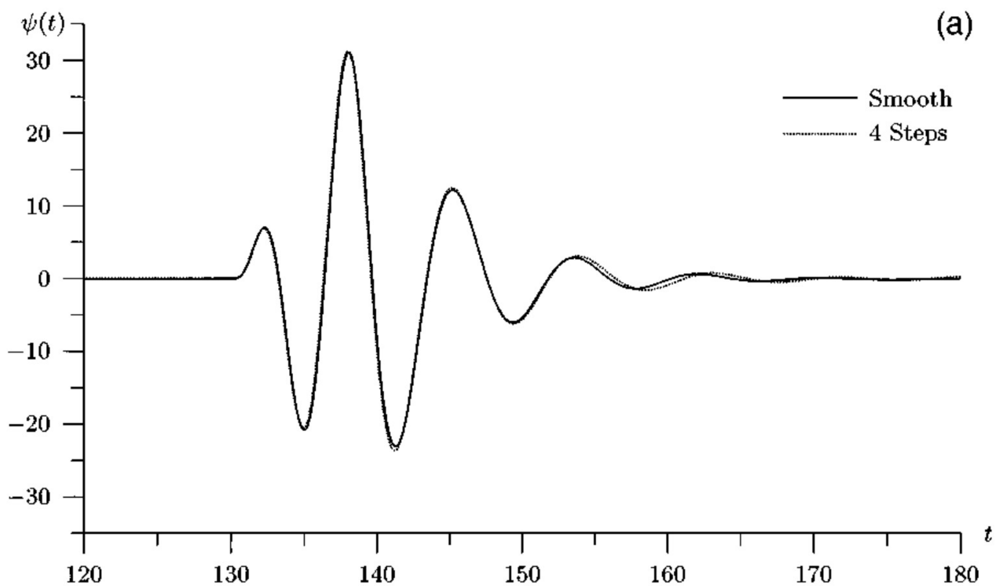
Nollert PRD 1996

Discretized potential





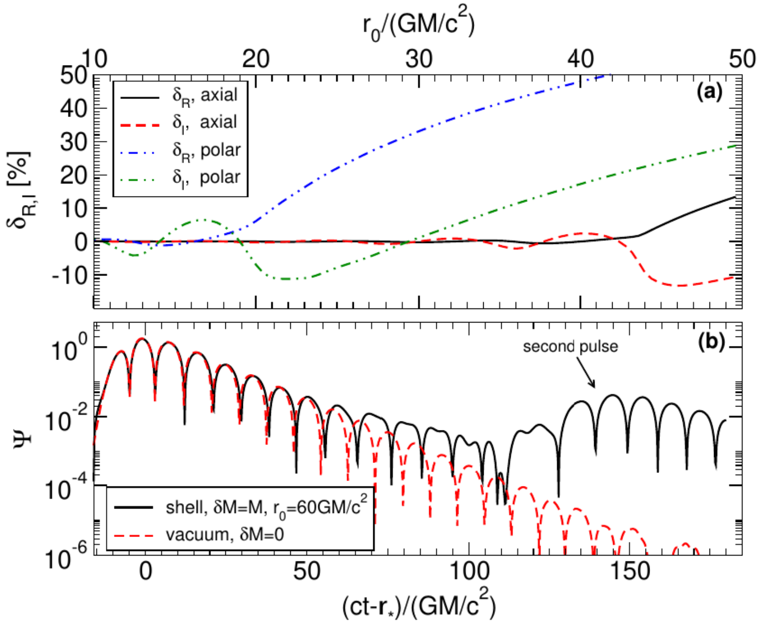
Different QNM spectrum, same time-domain ringdown



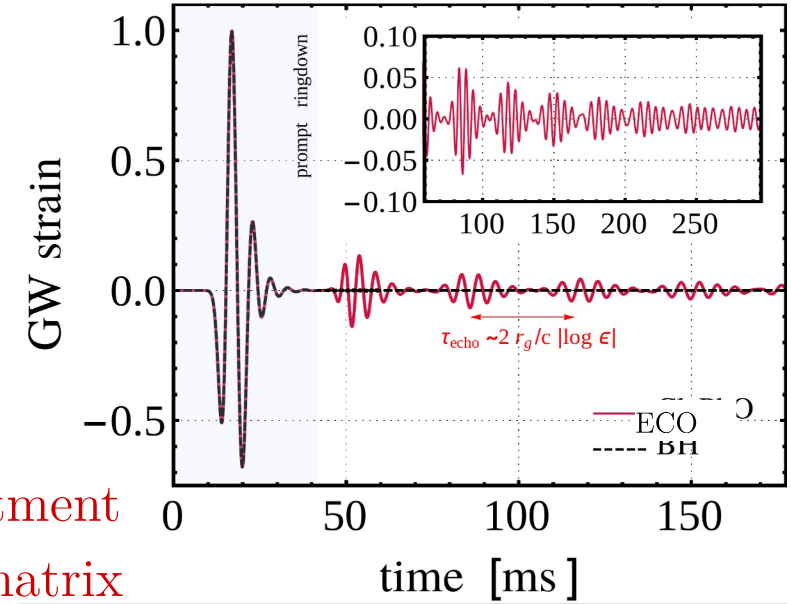
More QNM spectral instabilities

Environmental effects

Barausse, Cardoso & PP, PRD 2014



Near-horizon structure

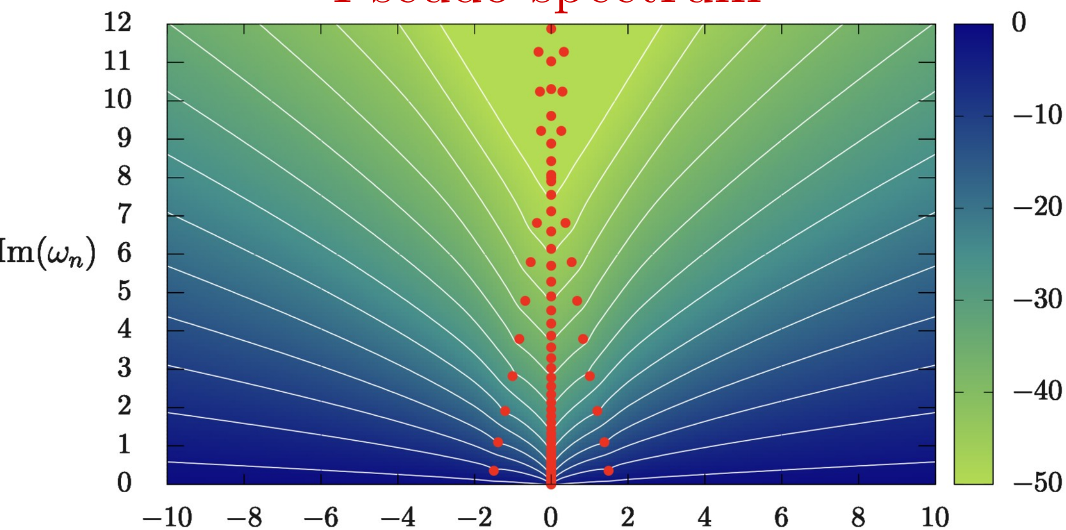


Cardoso & PP, Nature Astronomy (2017)

Analytical treatment with transfer matrix

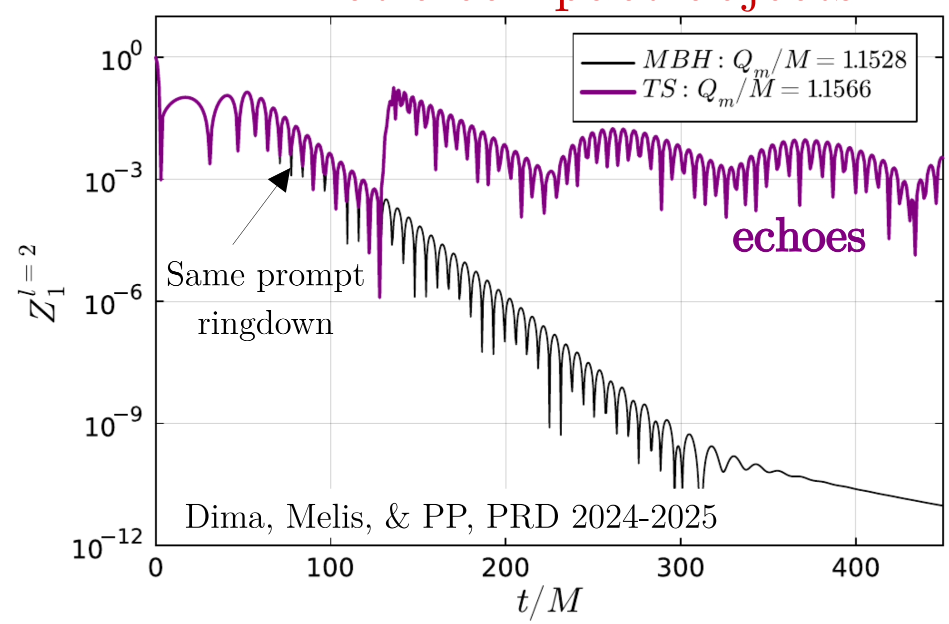
Ianniccari+ PRL 2024

Pseudo-spectrum



Jaramillo+ PRX 2020

Exotic compact objects



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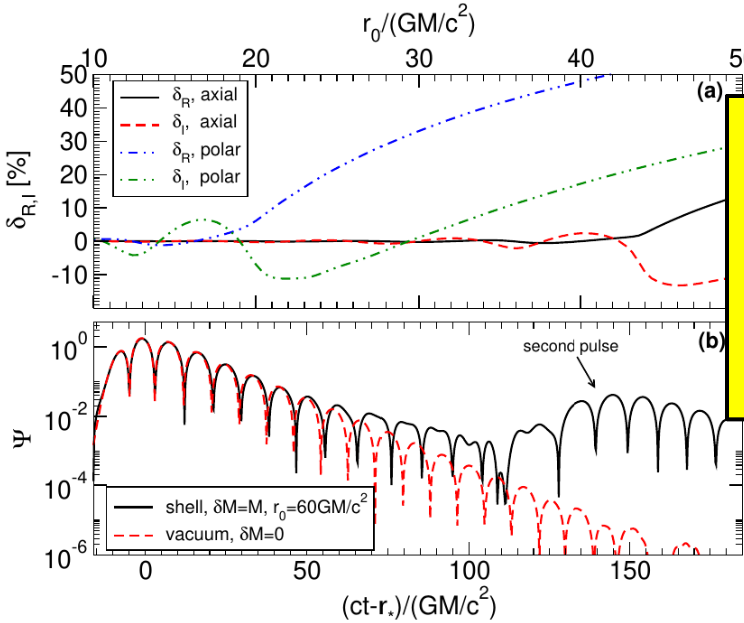

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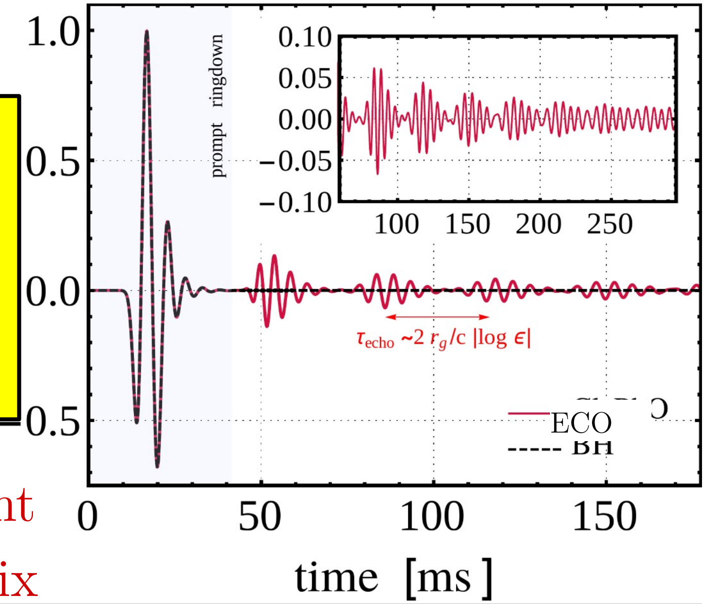

Cardoso & PP, Nature Astronomy (2017)

Initial ringdown **NOT** described by QNMs!

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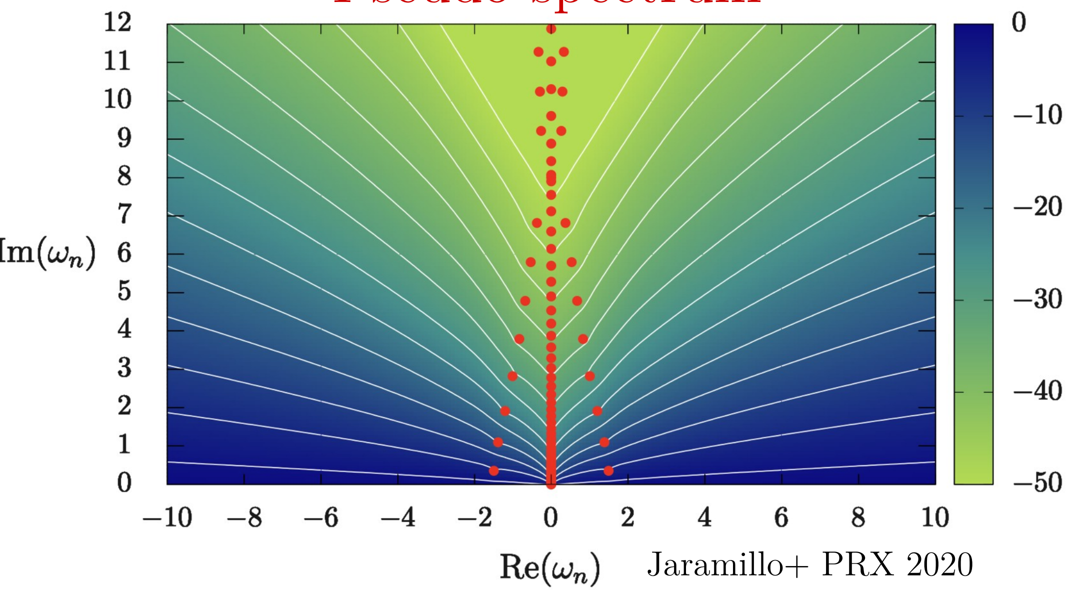

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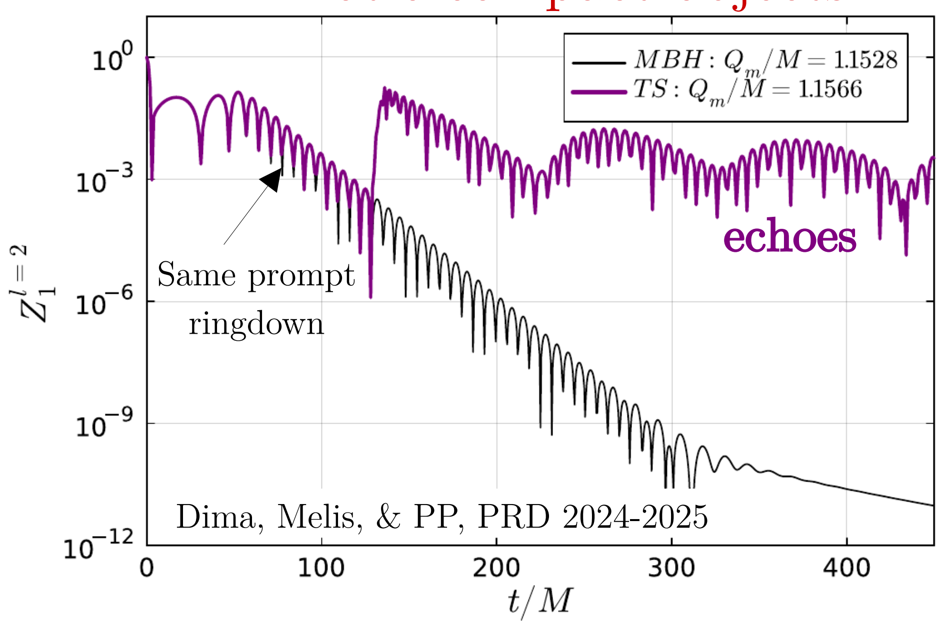
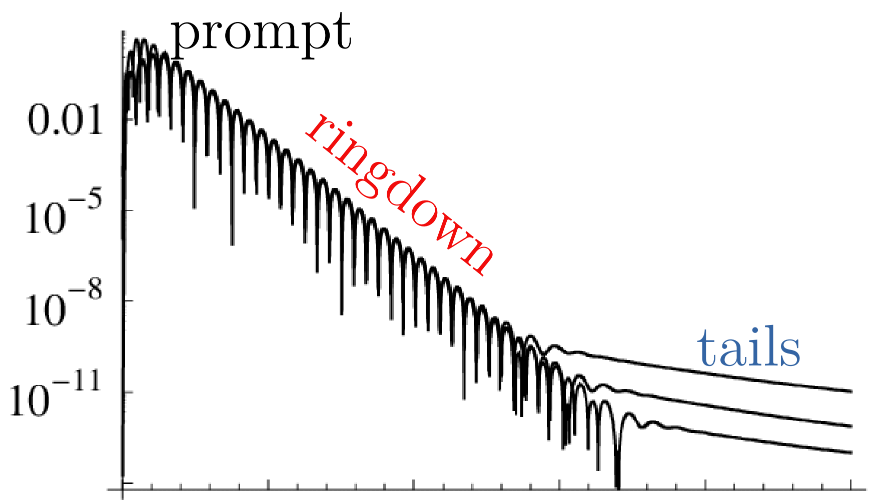


Dima, Melis, & PP, PRD 2024-2025

Take-home message

Even within linear perturbation theory:

- QNMs (as poles of Green's function) do not necessarily describe the time-domain signal $\rightarrow$ not directly measurable
- Time-domain signal: prompt response + ringdown + tails

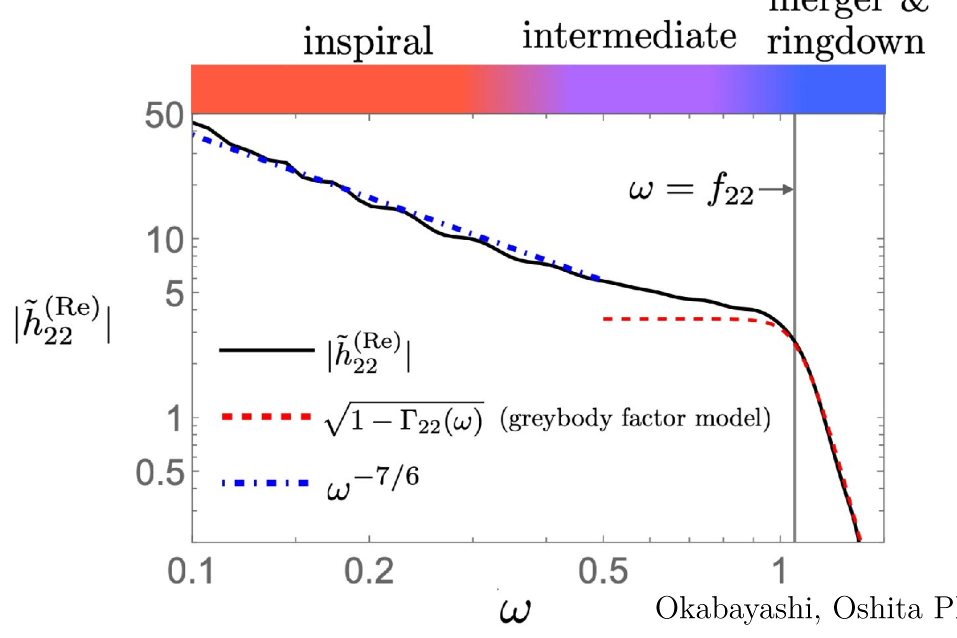


Are there other (stable) observable quantities characterizing the post-merger linear response?

Greybody factors as ringdown probes

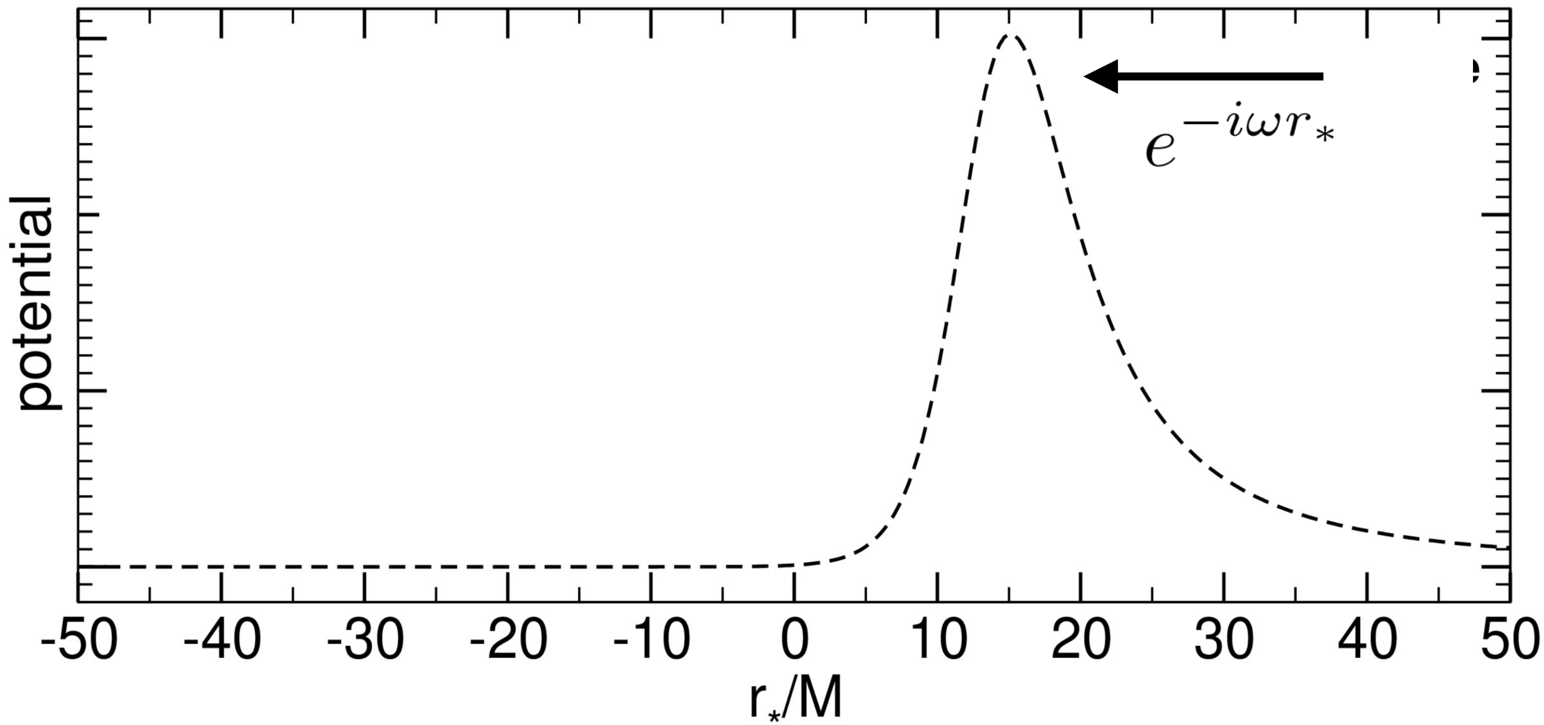
Oshita 2022-2024

GW spectral amplitude



Okabayashi, Oshita PRD 2024

Greybody factor (GF)



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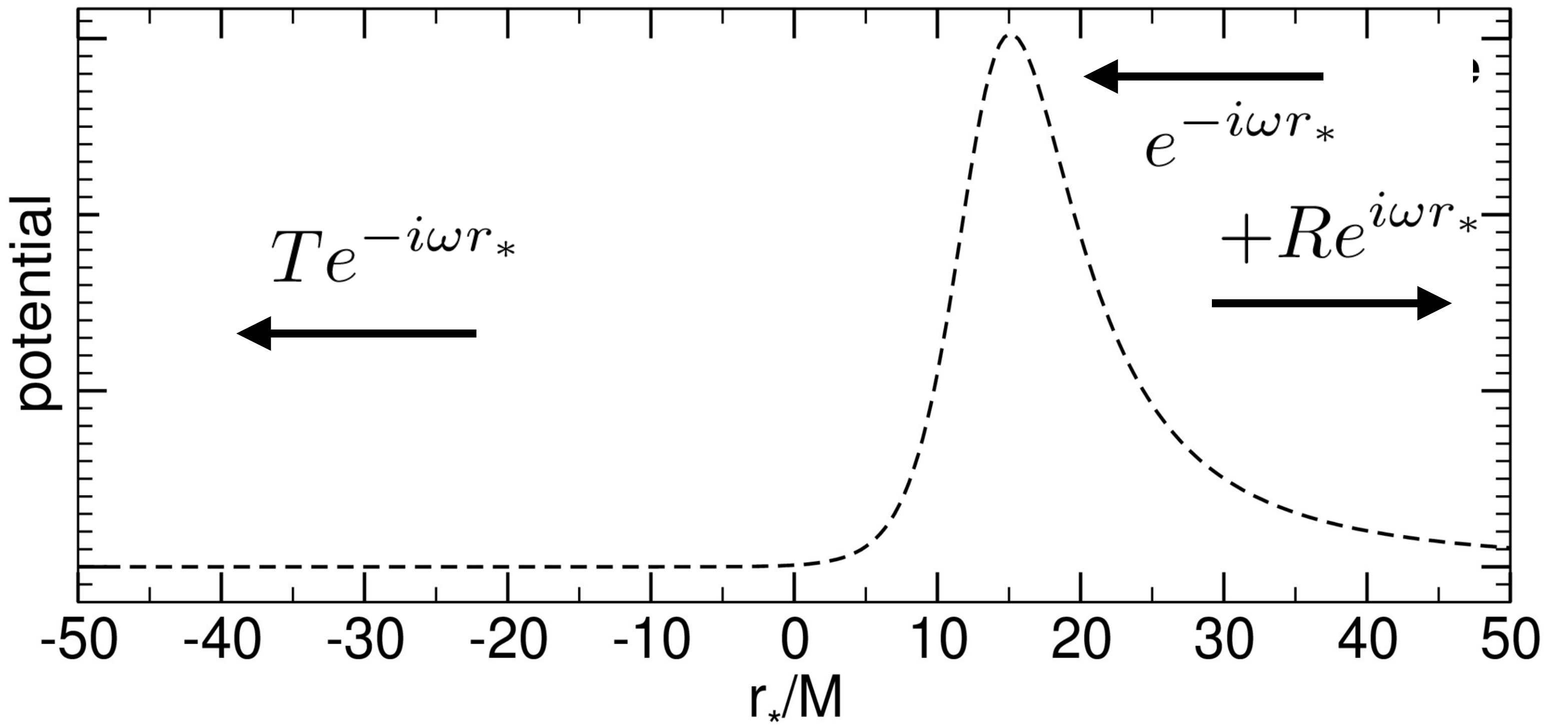

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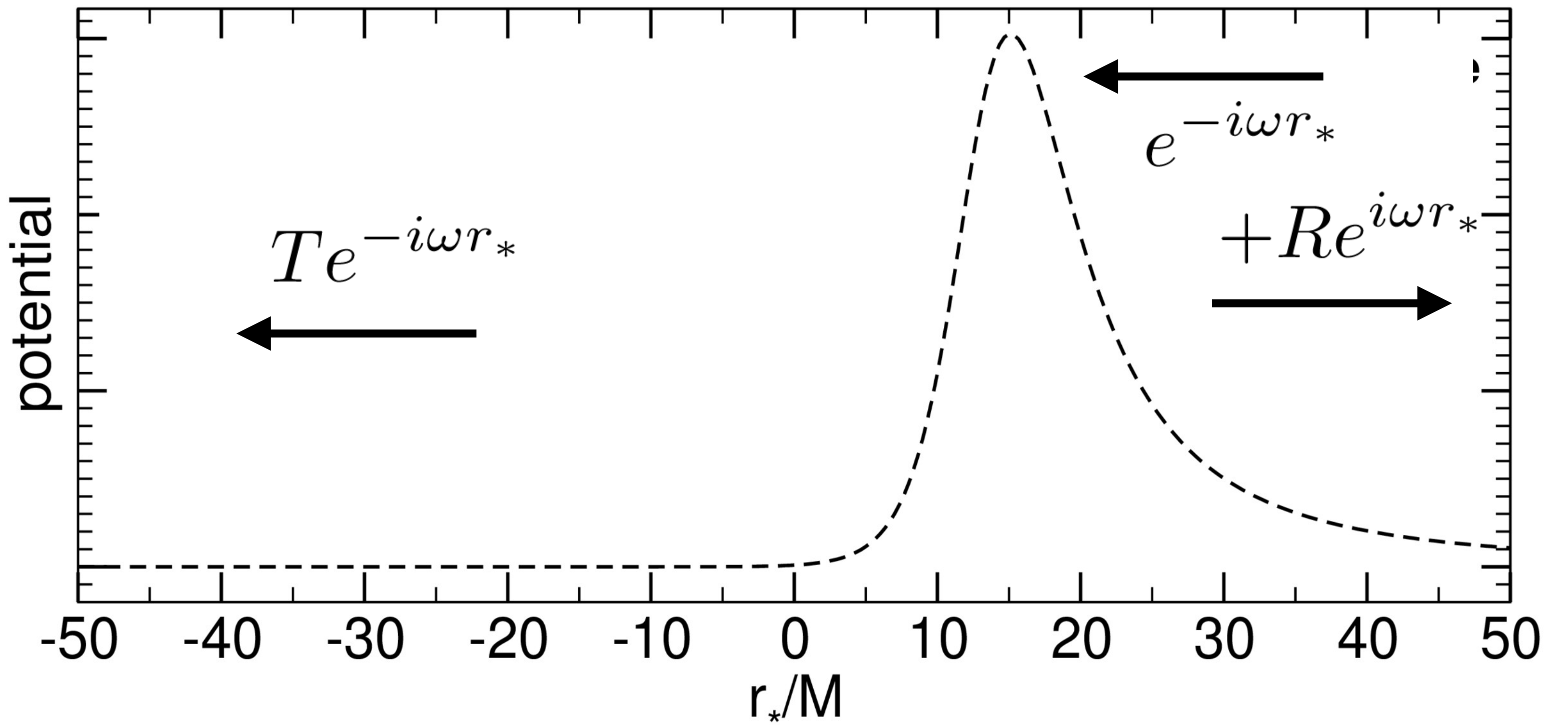


$$\Gamma = |T|^2$$

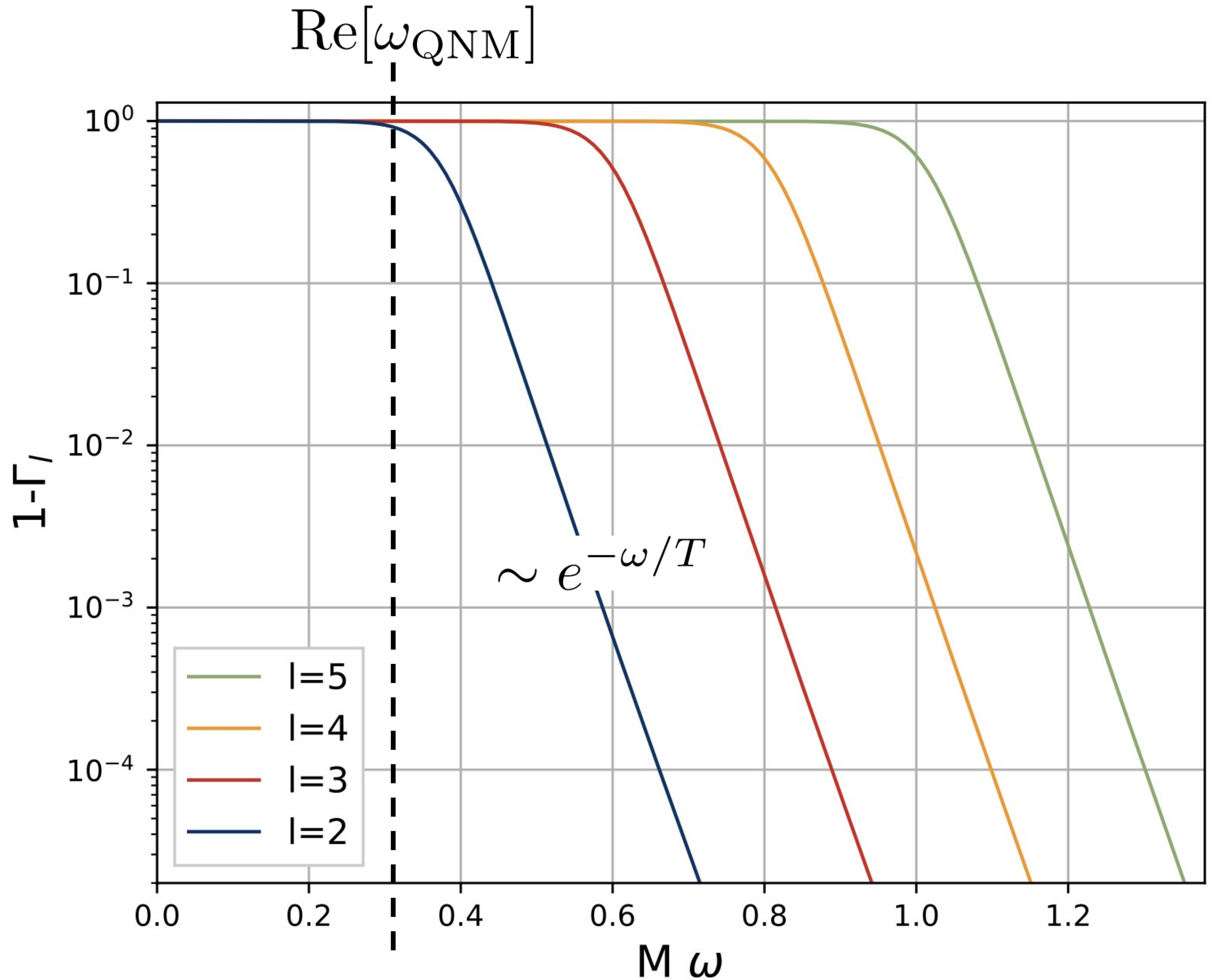
greybody factor
(transmittivity)

$$\mathcal{R} = |R|^2 = 1 - \Gamma$$

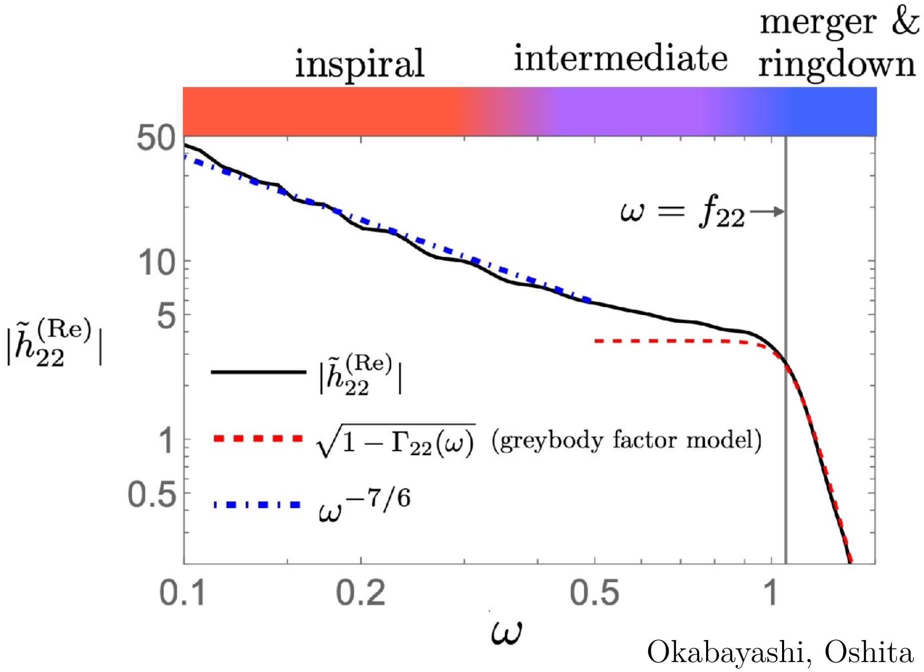
reflectivity

"no-hair" quantities

Greybody & Reflectivity



GF & ringdown spectral amplitude



Okabayashi, Oshita PRD 2024

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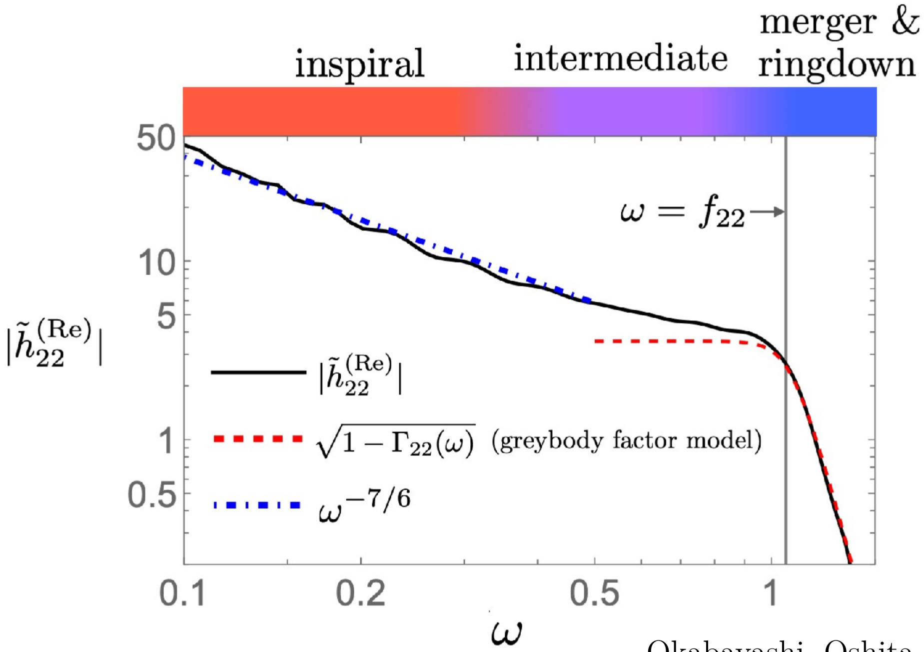
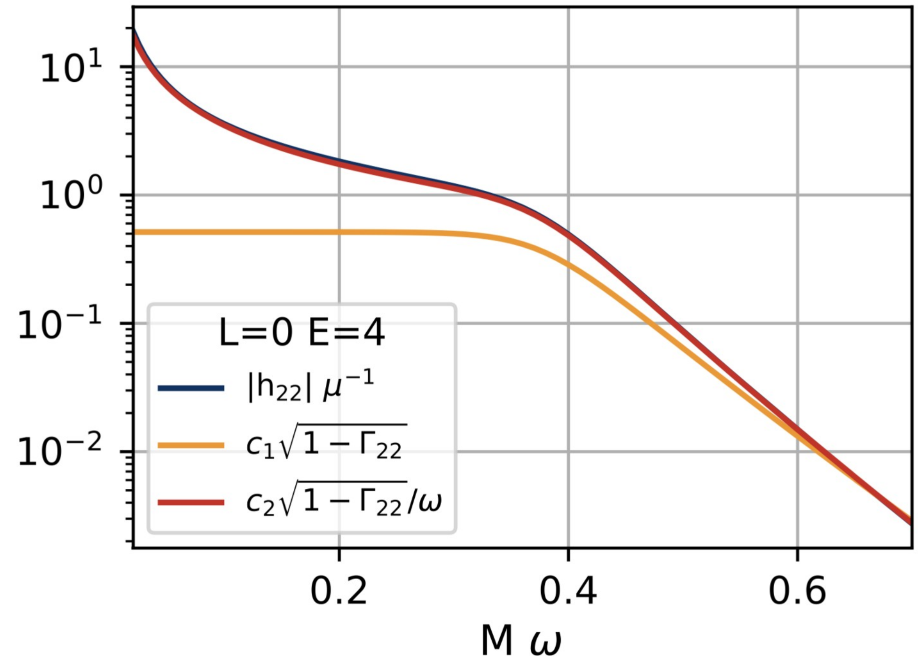
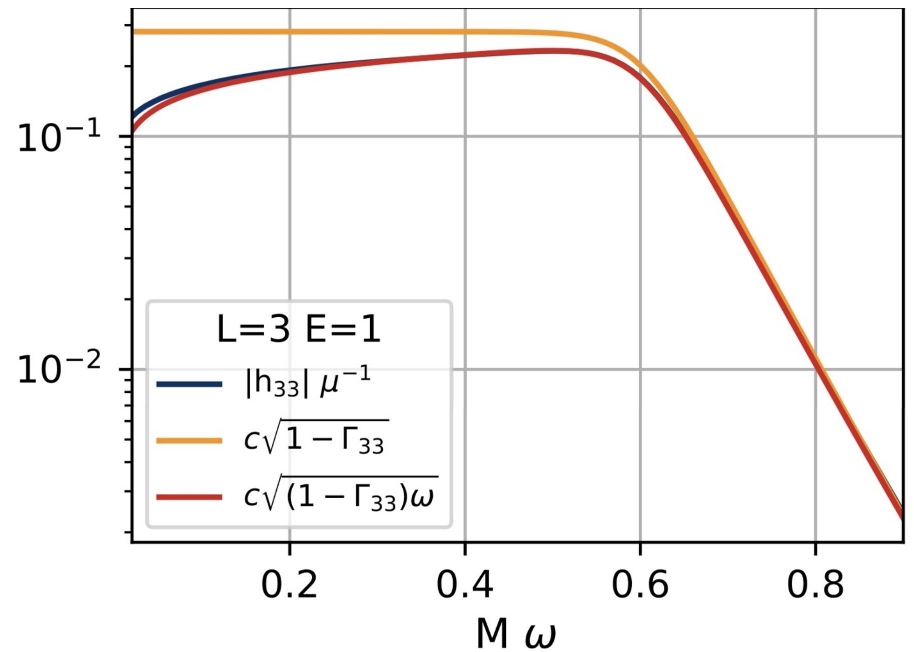


Okabayashi, Oshita PRD 2024



Works also at small frequencies!

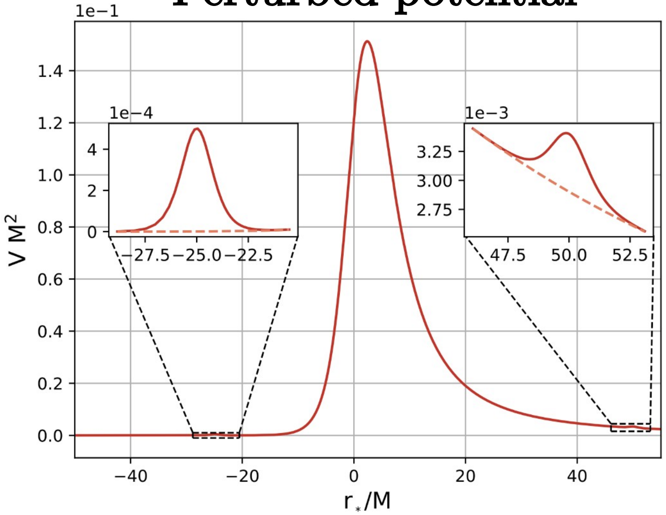


$$|h_{lm}(\omega)| \sim \frac{\sqrt{1-\Gamma_{lm}}}{\omega^p}$$

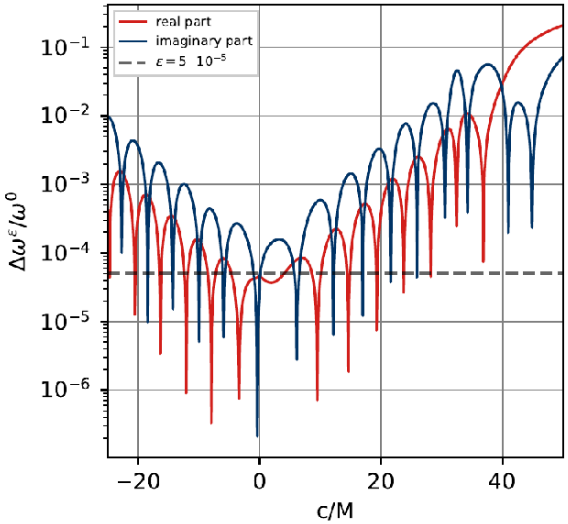
Rosato, Destounis, Pani
Phys.Rev.D 110 (2024) 12, L121501

Stability of the GF

Perturbed potential



QNM spectral instability



Rosato+ PRD 2024-2025

Oshita, Takahashi, Mukohyama PRD 2024

Stability of the GF

Rosato+ PRD 2024-2025
Oshita, Takahashi, Mukohyama PRD 2024

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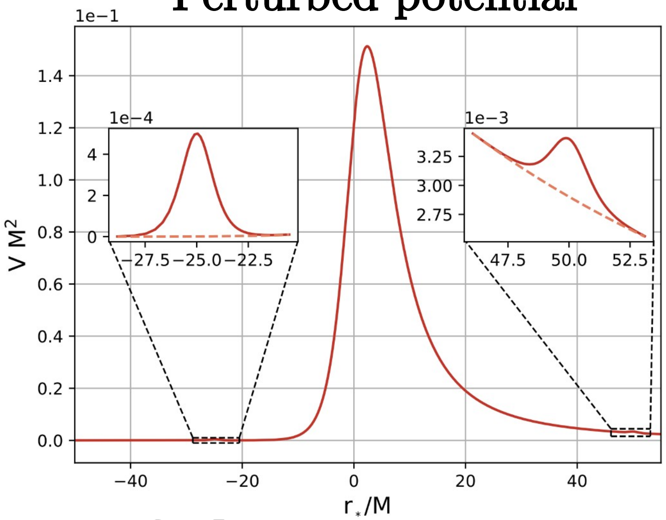

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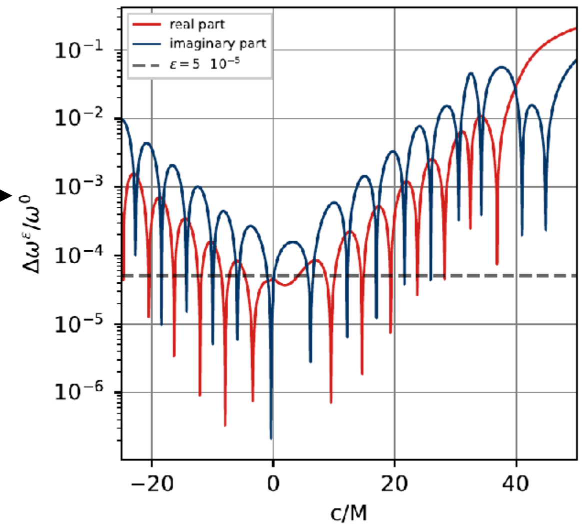

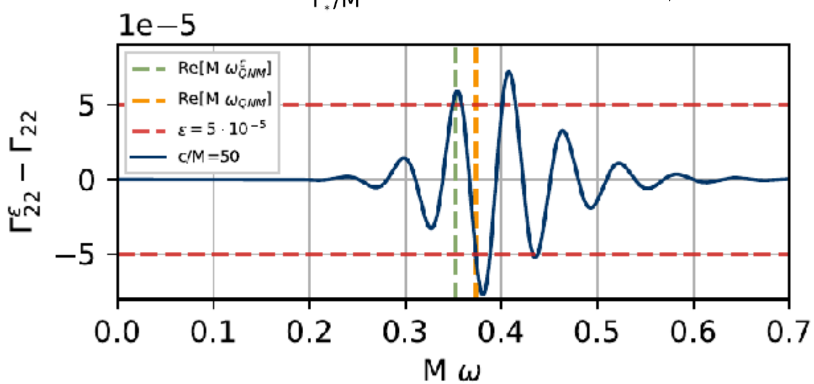

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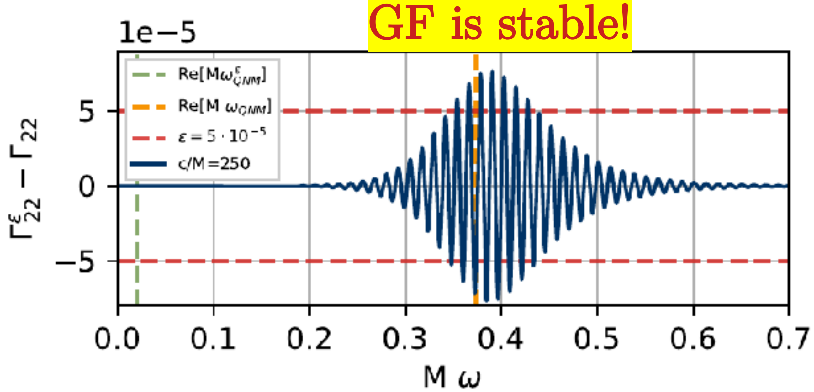

Stability of the GF

Rosato+ PRD 2024-2025
Oshita, Takahashi, Mukohyama PRD 2024

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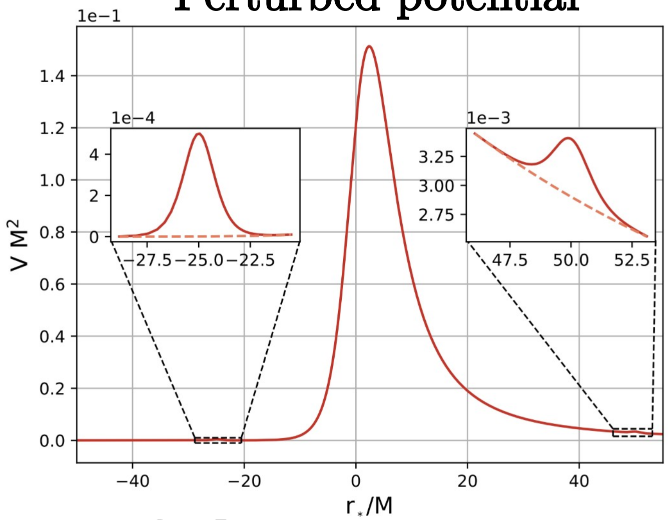

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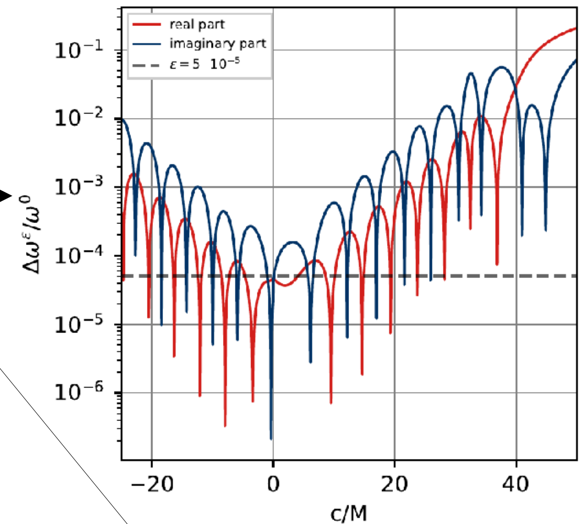

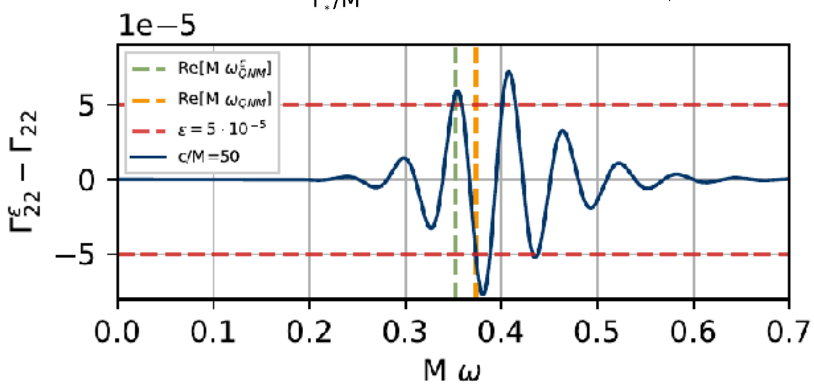

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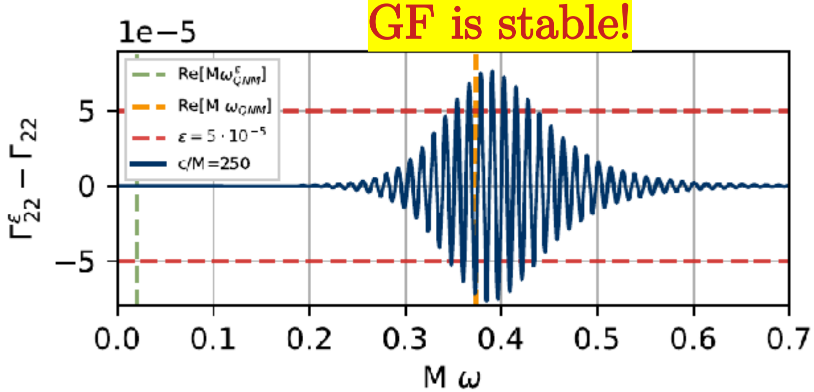
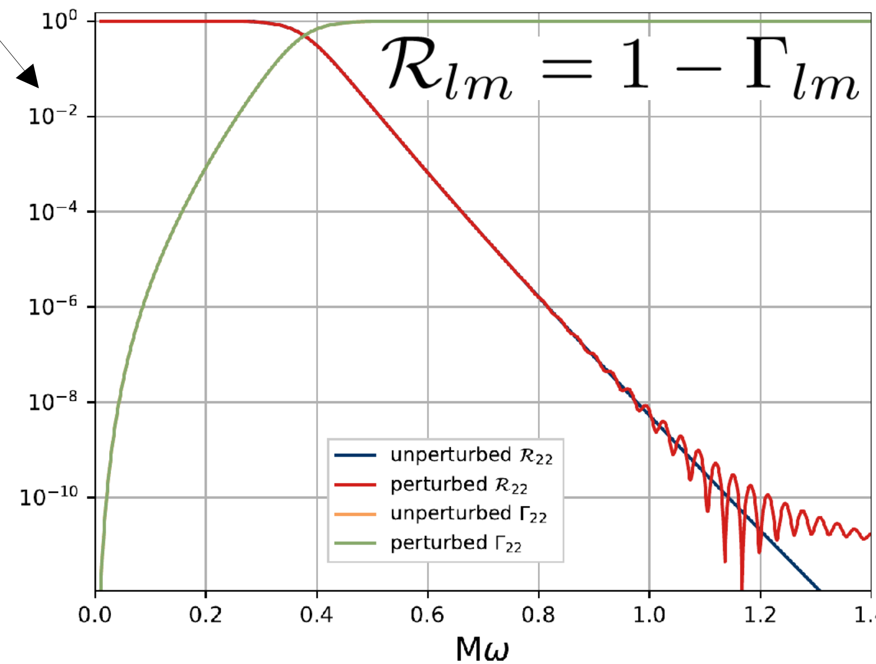
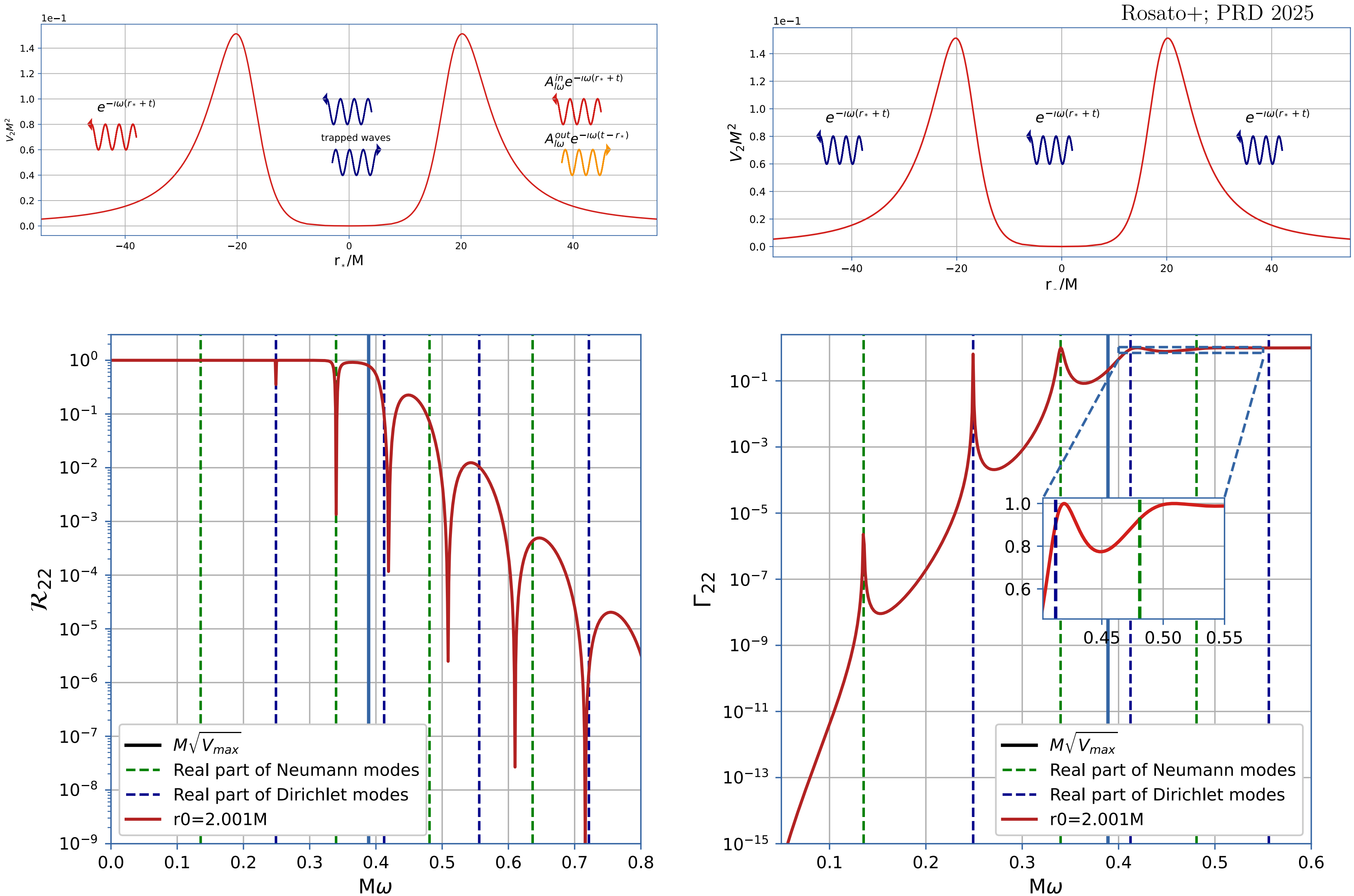




High-freq changes in reflectivity

GFs of horizonless objects

Rosato+; PRD 2025

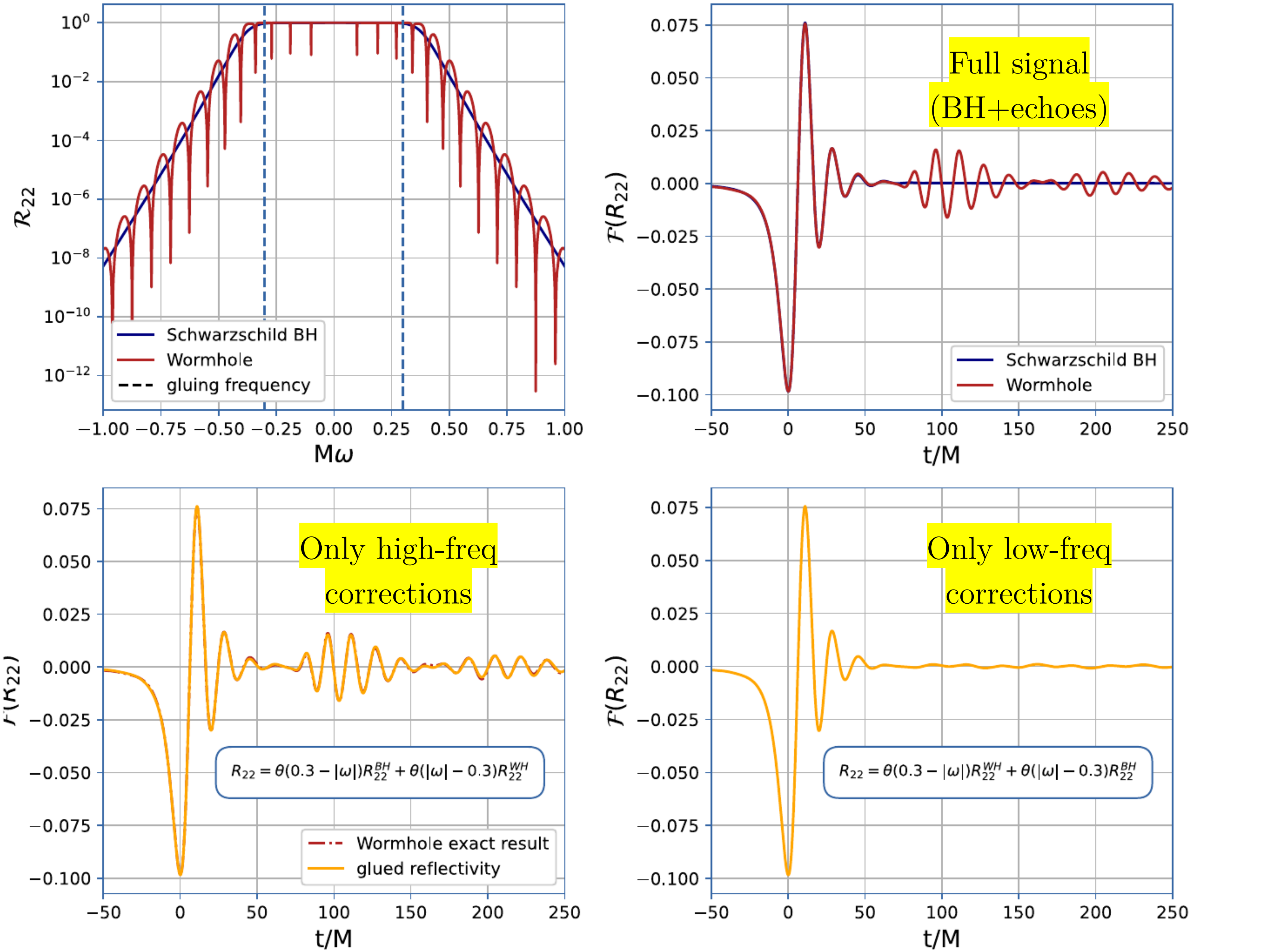


High-freq oscillations
& reflectionless scattering modes

Low-freq Breit-Wigner resonances
≡ QNMs

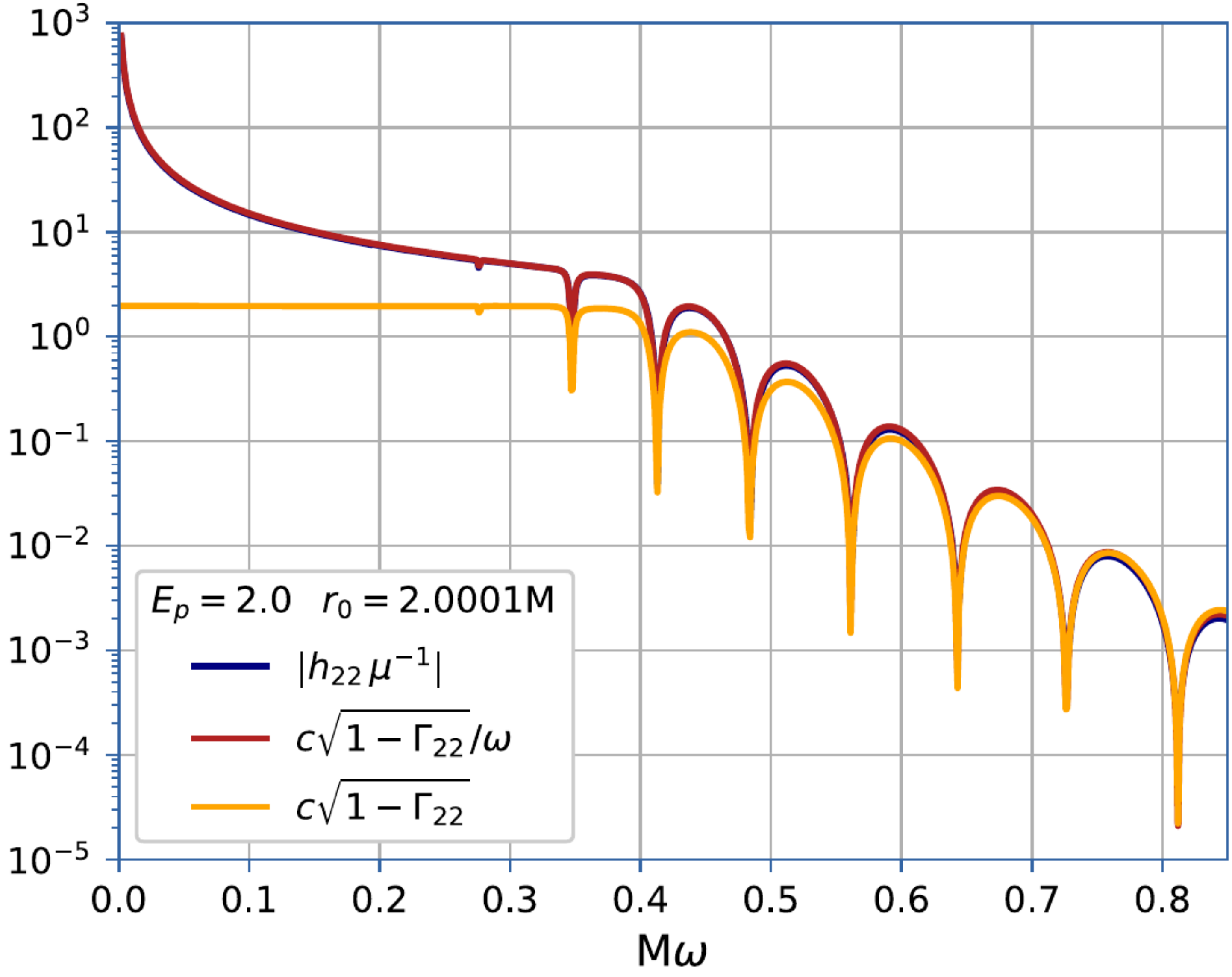
HF oscillations → Echoes

Rosato+; PRD 2025



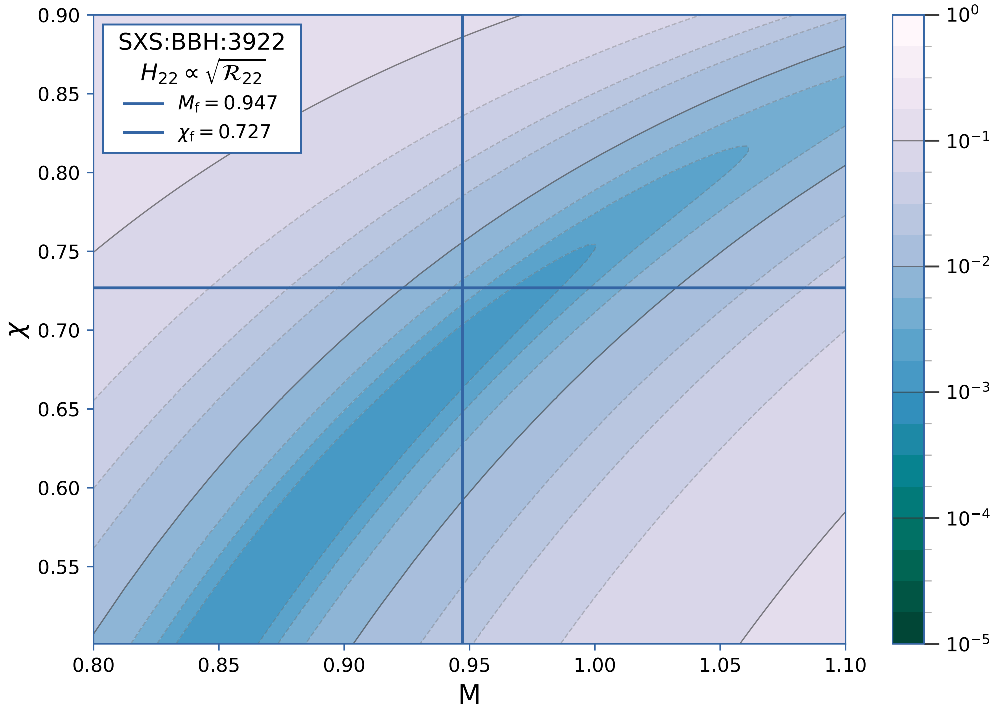
A perfect match: point particle

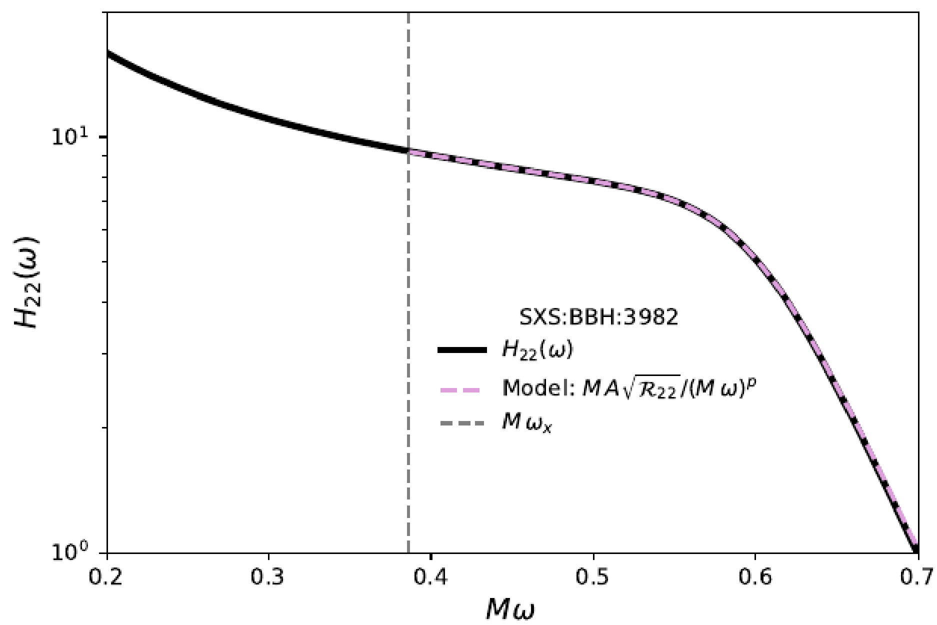
Rosato+; PRD 2025

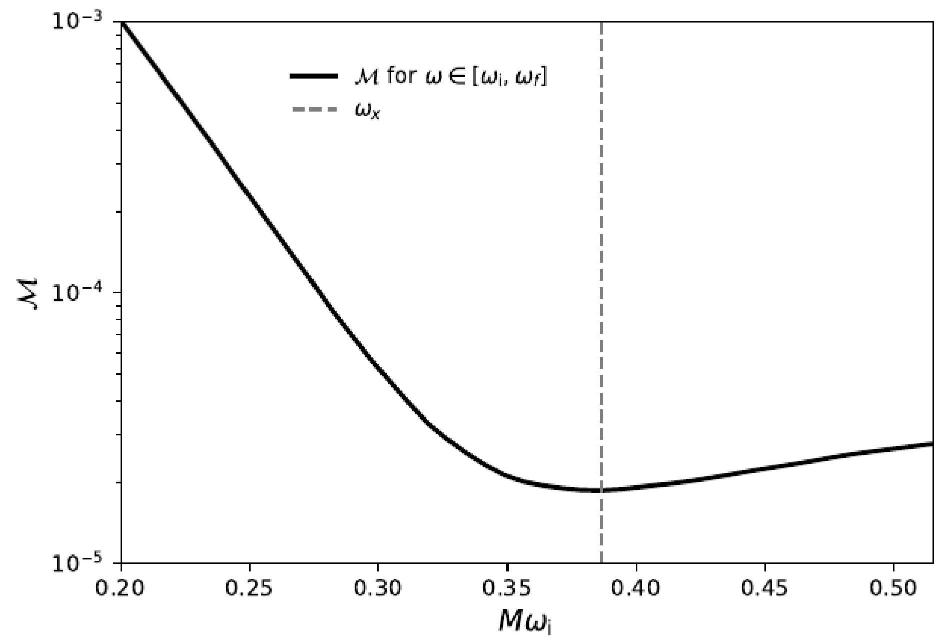


A perfect match: NR simulations

Rosato, Yi, Berti, Pani; Phys.Rev.D 113 (2026) 6, 064060







Simple 4-parameter model!

$$|h_{\ell m}(\omega)| = M \frac{A_{\ell m}\sqrt{\mathcal{R}_{\ell m}(M\omega, \chi)}}{(M\omega)^{p_{\ell m}}}$$

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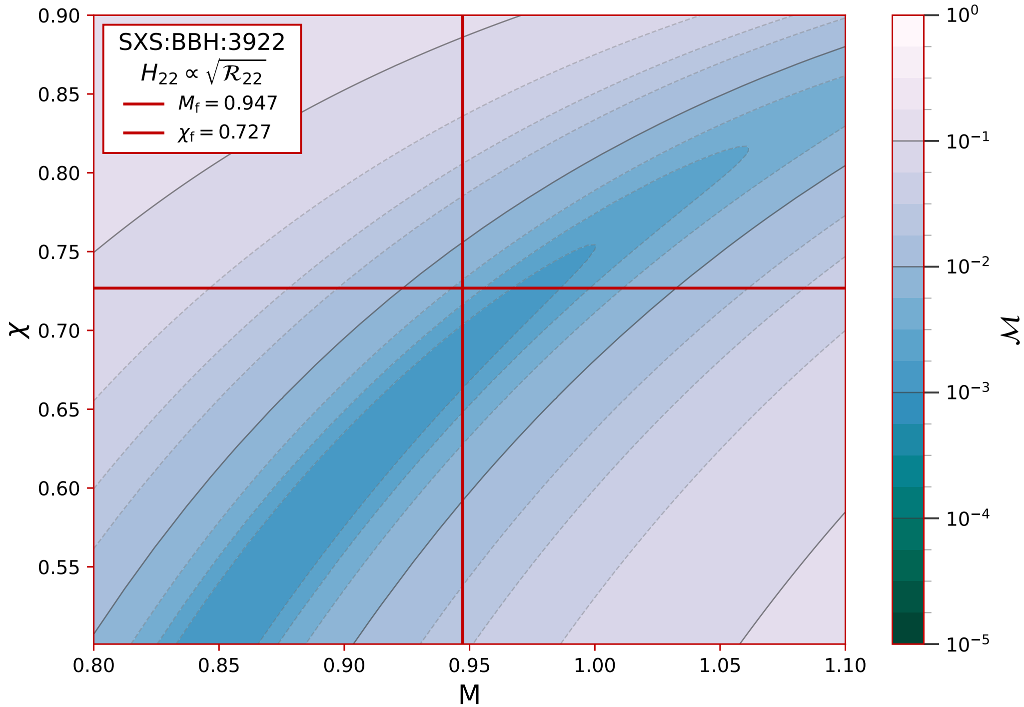

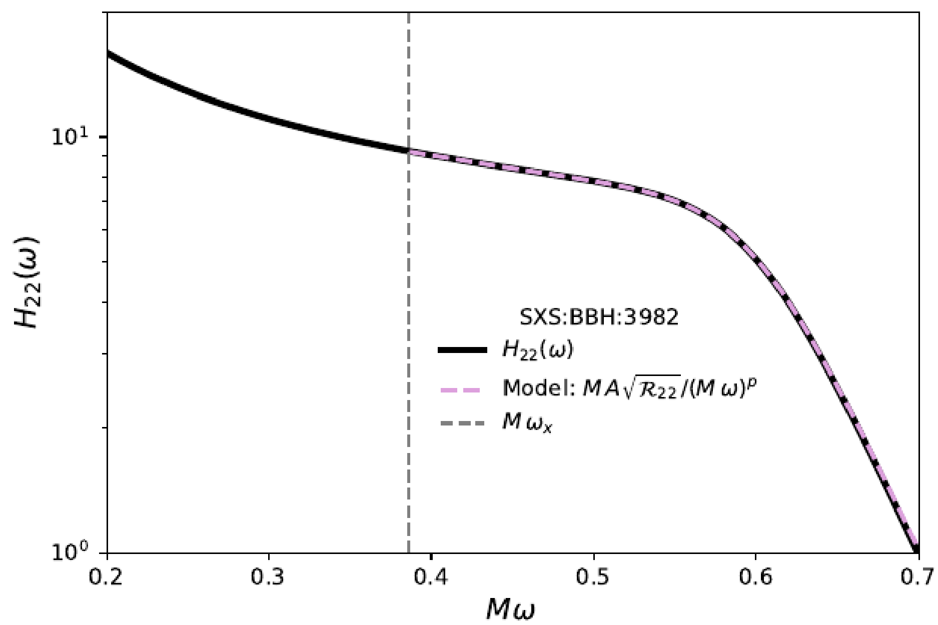

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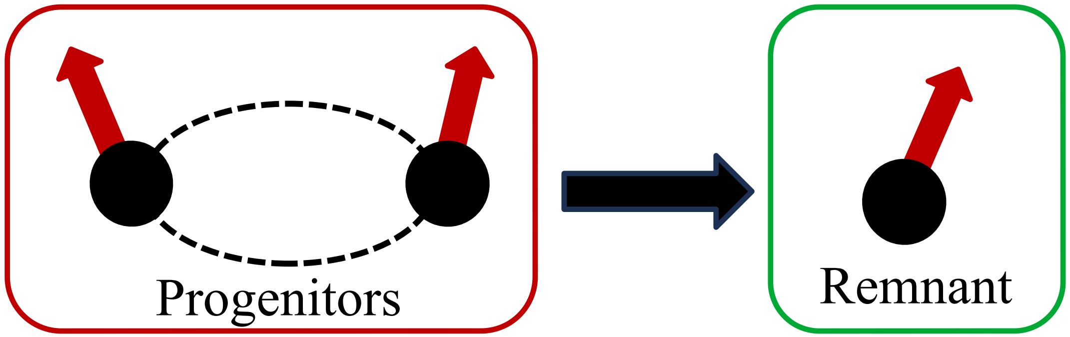
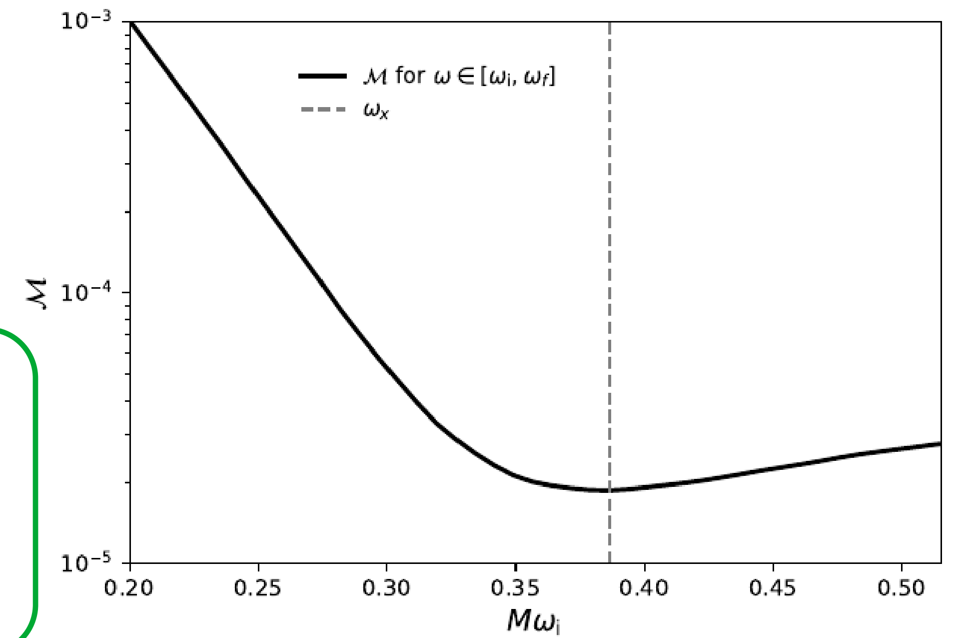
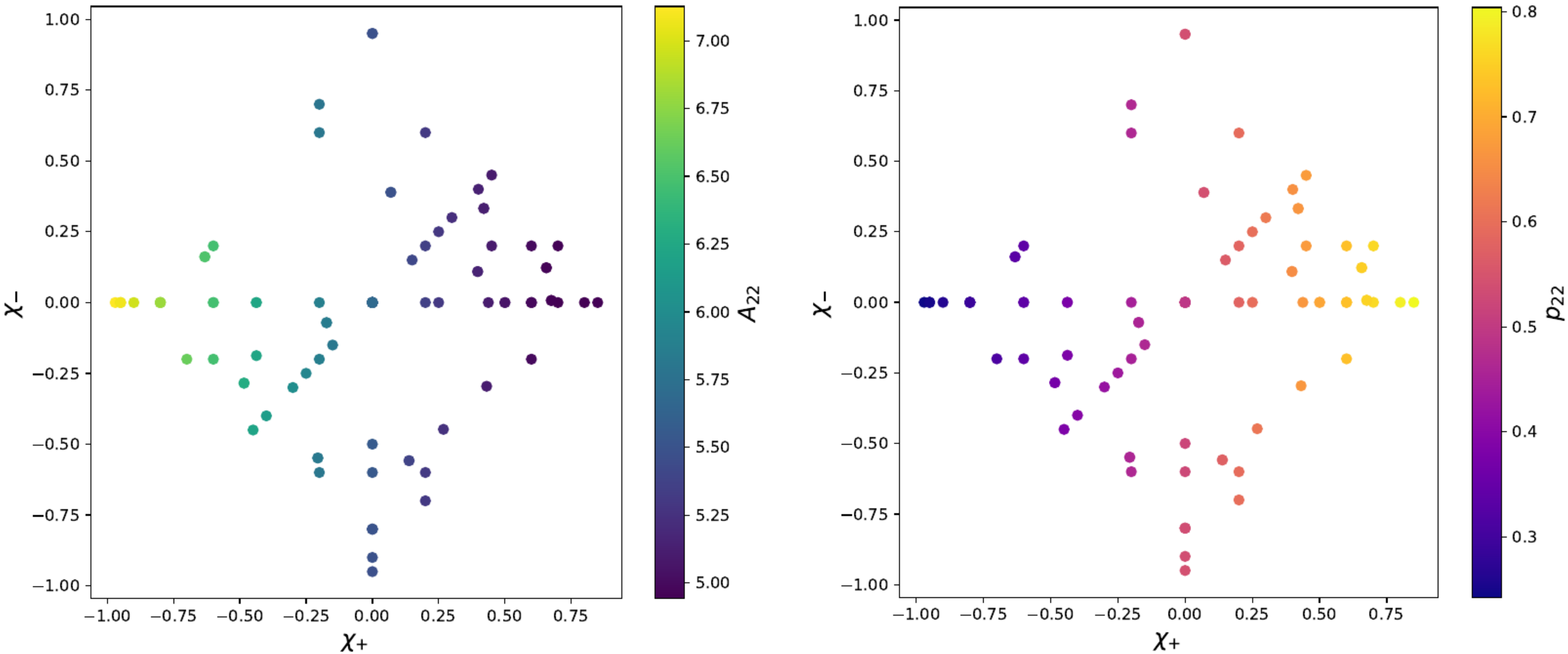




Dependence on progenitor

Rosato, Yi, Berti, Pani; Phys.Rev.D 113 (2026) 6, 064060



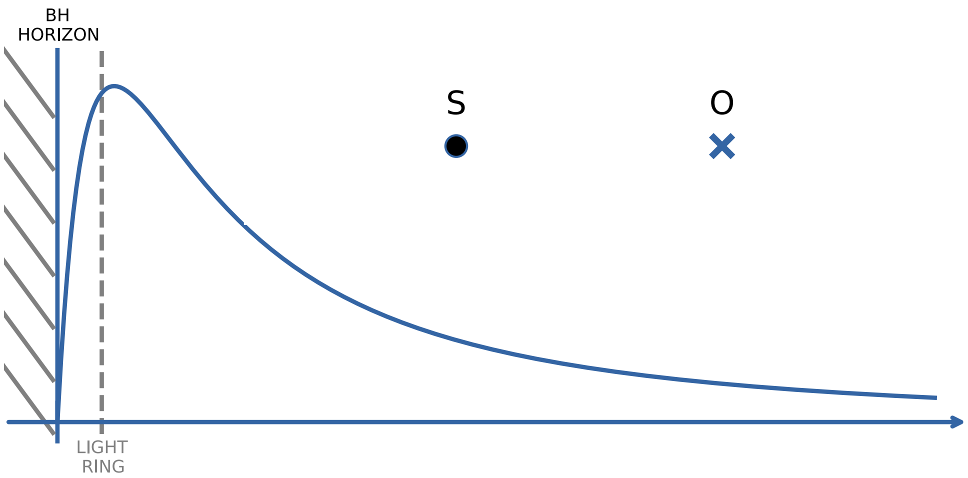
$$|h_{\ell m}(\omega)| = M\frac{A_{\ell m}\sqrt{\mathcal{R}_{\ell m}(M\omega,\chi)}}{(M\omega)^{p_{\ell m}}}$$

Fits available for (2,2), (2,1), (3,3), (4,4)

Typical mismatch~10^{-4} !!

Why Greybody in the waveform?

Rosato, De Amicis, Pani; 2603.20490



$$\left[\frac{d^2}{dr_*^2} + \omega^2 - V_\ell^{e/o}(r)\right] \tilde{\Psi}^{e/o}_{\ell m\omega}(r) = \tilde{S}^{e/o}_{\ell m}(\omega, r)$$

$$u^{\rm in}_{\ell m\omega} \to \begin{cases} A^{\rm in}_{\ell m\omega}\, e^{-i\omega r_*} + A^{\rm out}_{\ell m\omega}\, e^{i\omega r_*}, & r_* \to +\infty, \\ e^{-i\omega r_*}, & r_* \to -\infty, \end{cases}$$

$$u^{\rm down}_{\ell m\omega} \to \begin{cases} e^{-i\omega r_*}, & r_* \to +\infty, \\ B^{\rm in}_{\ell m\omega}\, e^{-i\omega r_*} + B^{\rm out}_{\ell m\omega}\, e^{i\omega r_*}, & r_* \to -\infty\,, \end{cases} \qquad u^{\rm up}_{\ell m\omega} = u^{\rm down}_{\ell m,-\omega}$$

$$u^{\rm in}_{\ell m\omega} = A^{\rm in}_{\ell m\omega}\, u^{\rm down}_{\ell m\omega} + A^{\rm out}_{\ell m\omega}\, u^{\rm up}_{\ell m\omega}$$

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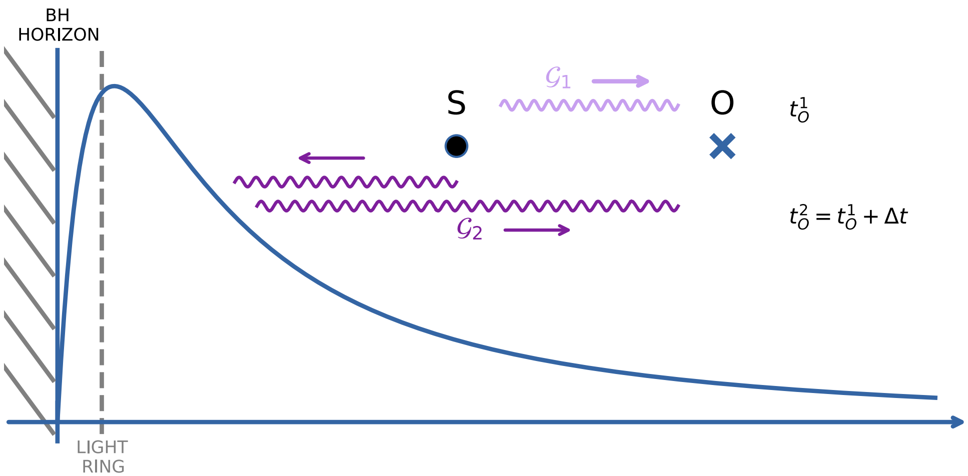

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Green's function:

$$\tilde{G}_{\ell m}(r_*, r'_*, \omega) = \boxed{\frac{i\, e^{i\omega r_*}}{2\omega}\, u^{\rm down}_{\ell m\omega}(r'_*)} + \boxed{\frac{A^{\rm out}_{\ell m\omega}}{A^{\rm in}_{\ell m\omega}} \frac{i\, e^{i\omega r_*}}{2\omega}\, u^{\rm up}_{\ell m\omega}(r'_*)}$$

G_1, direct — G_2, scattered

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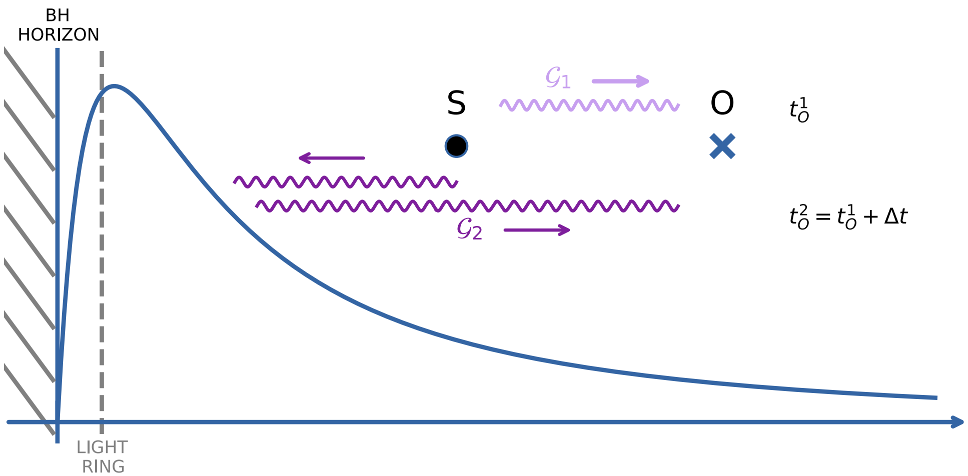

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G_1, direct

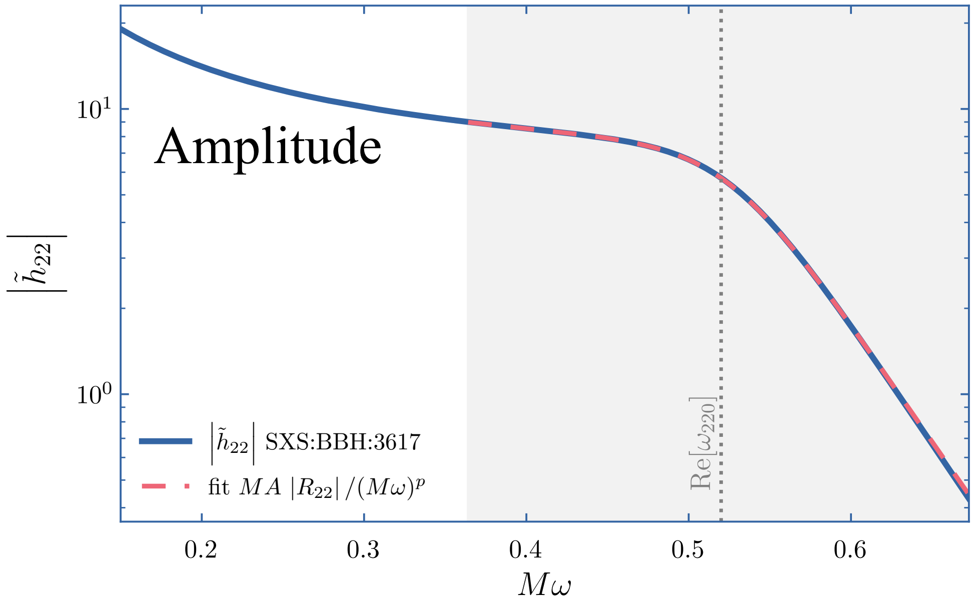
G_2, scattered

Reflection coefficient!

(~sqrt(reflectivity))

Full GreyRing model

$$|\tilde{h}_{\ell m}(\omega)| = M \frac{A_{\ell m}}{(M\omega)^{p_{\ell m}}} |R_{\ell m}(M\omega, \chi)|$$



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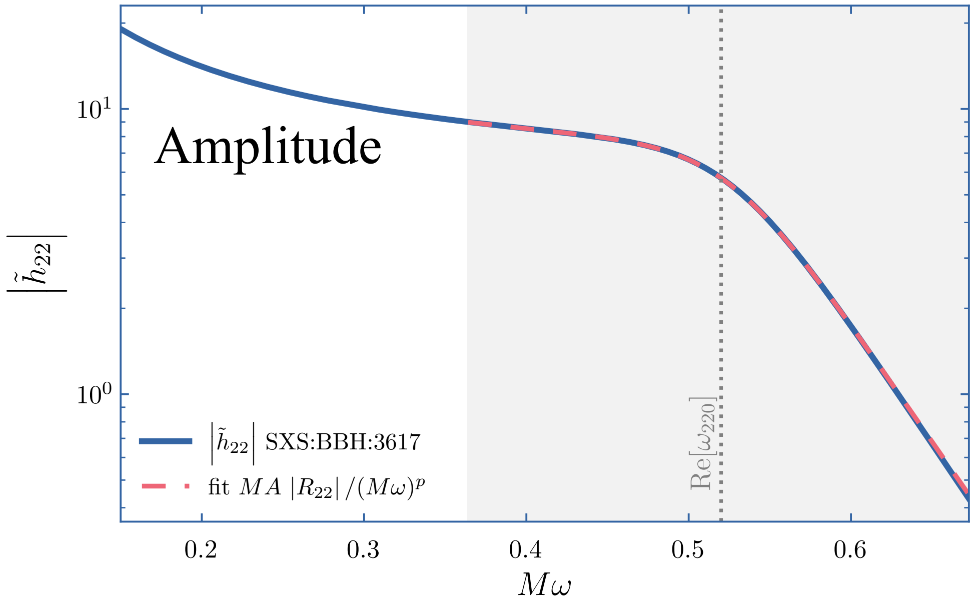
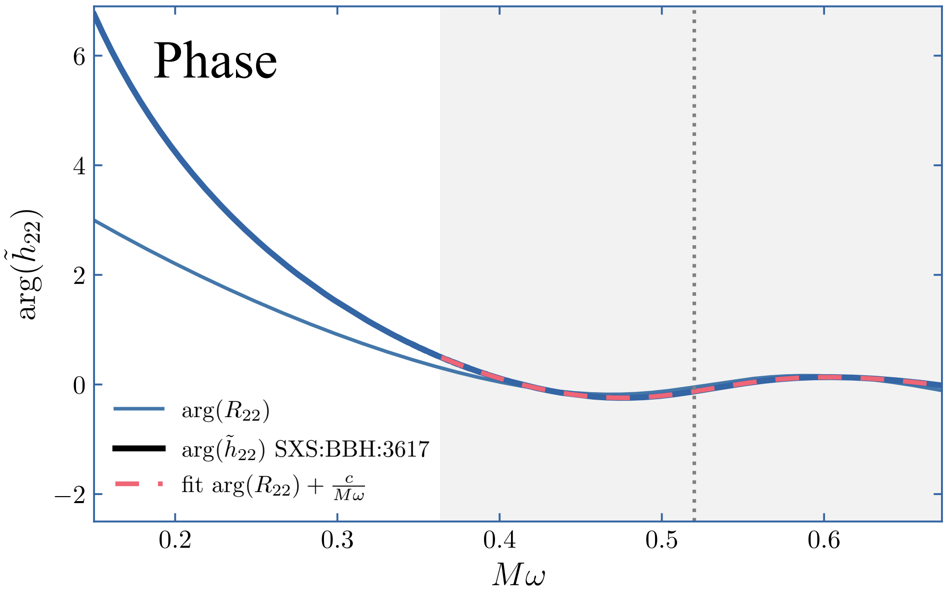


$$\arg\left(\tilde{h}_{\ell m}(\omega)\right) = \arg(R_{\ell m}(M\omega,\chi)) + \omega t_c + \phi_c + \frac{c_{\ell m}}{M\omega}$$



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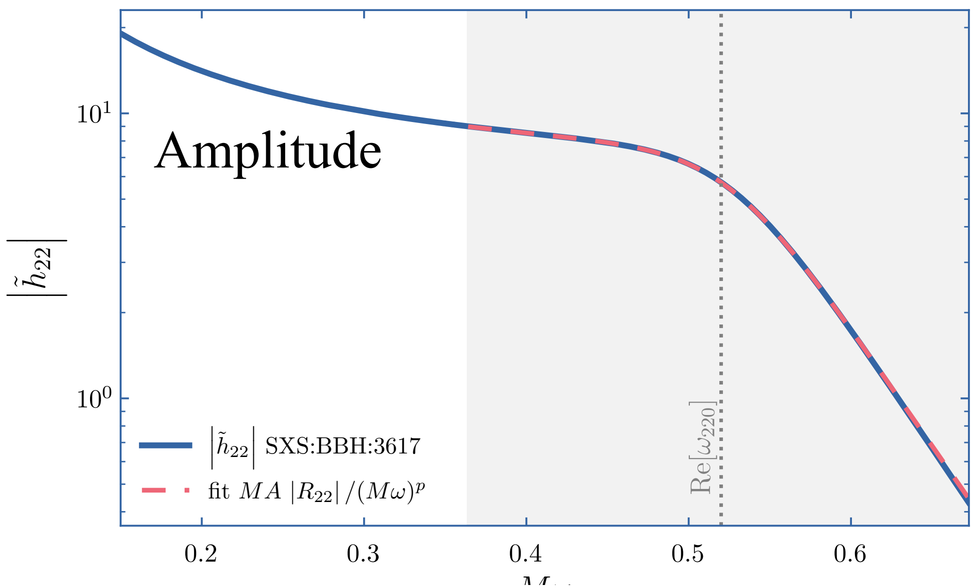
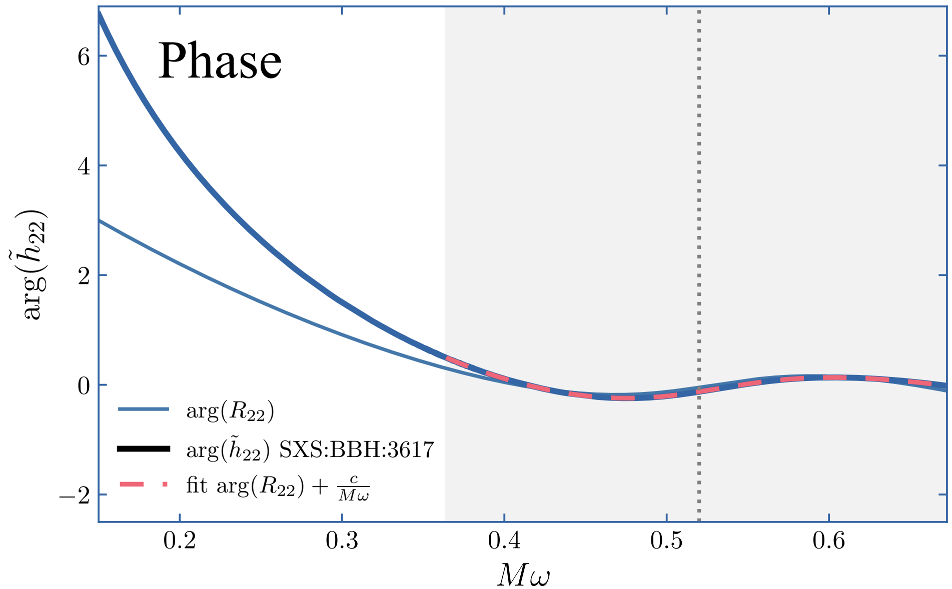
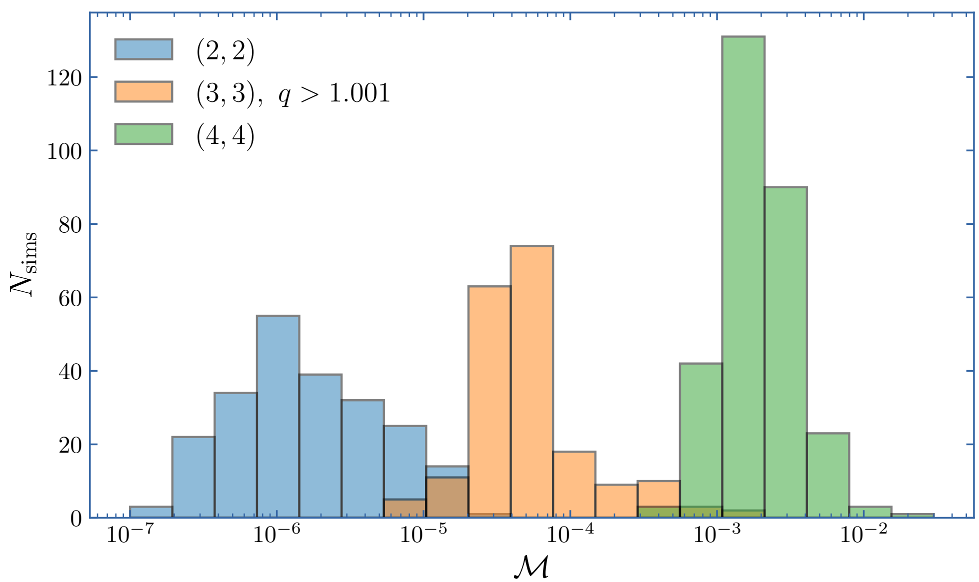


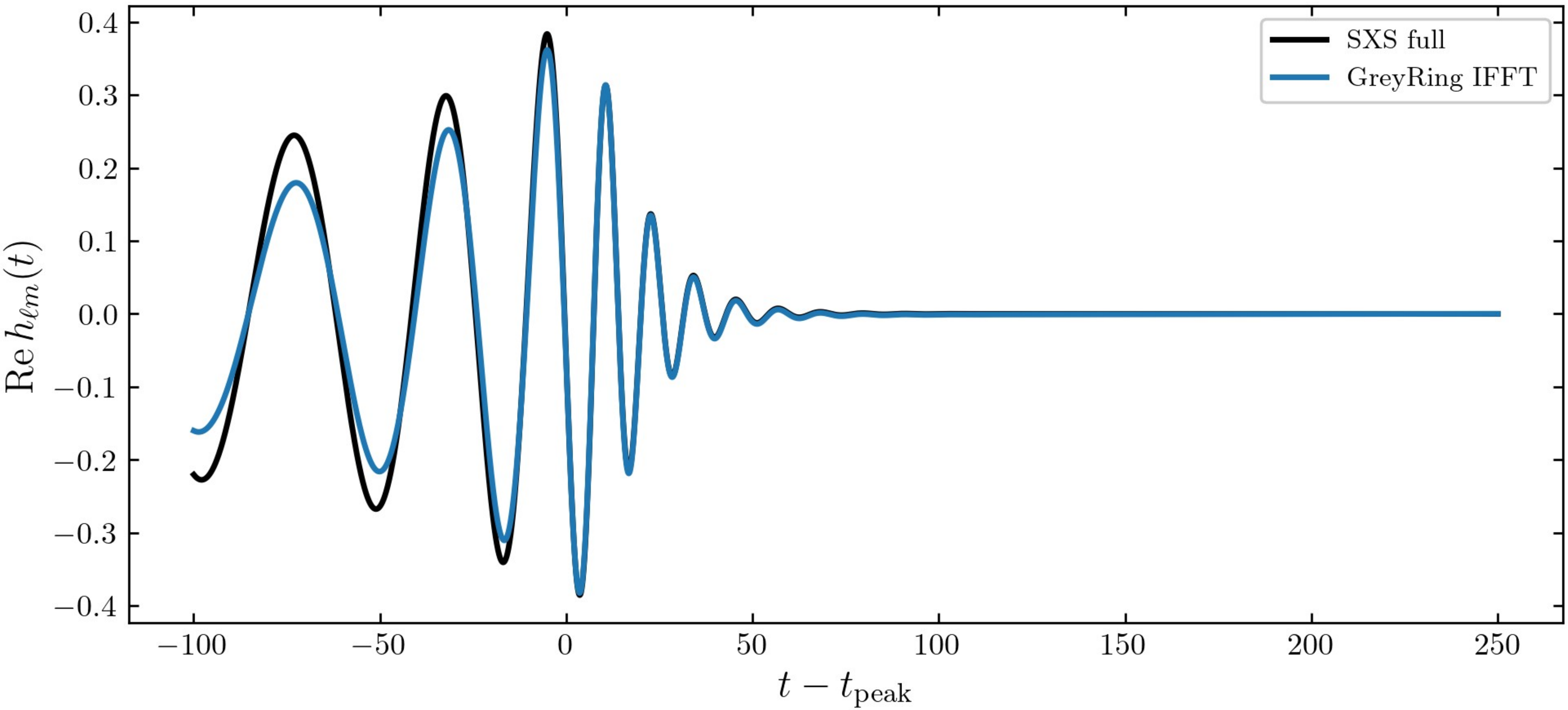




$$\mathcal{M} = 1 - \frac{\langle H^{\rm data} | H^{\rm model} \rangle}{\sqrt{\langle H^{\rm data} | H^{\rm data} \rangle \langle H^{\rm model} | H^{\rm model} \rangle}}$$

mismatch

Back to time domain

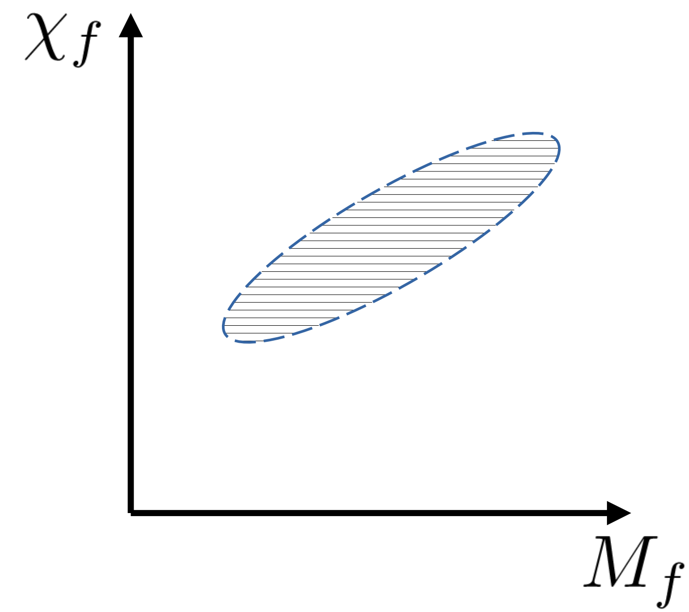


Novel test of GR with GreyRing

Rosato+; 2604.11895

$$h(t) \rightarrow (f_{220}, \tau_{220}) \rightarrow (M_f, \chi_f)$$

spectroscopy



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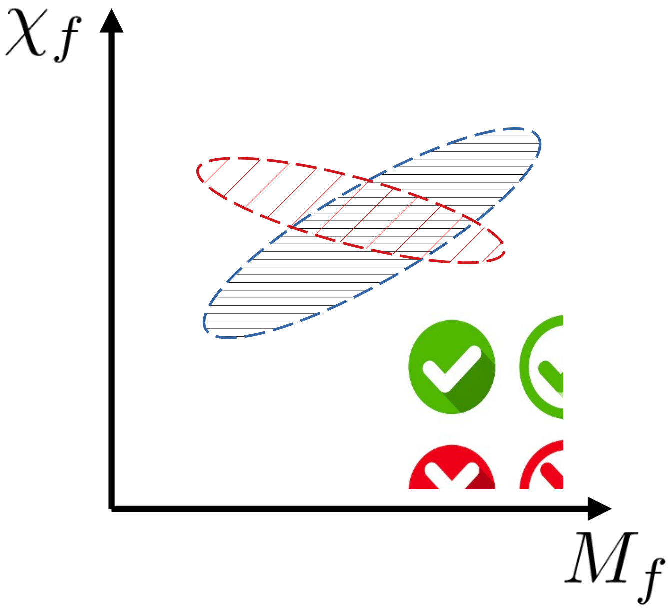
spectroscopy

$$\tilde{h}(\omega) \to R_{22}(\omega) \to (M_f, \chi_f)$$

greybody factor



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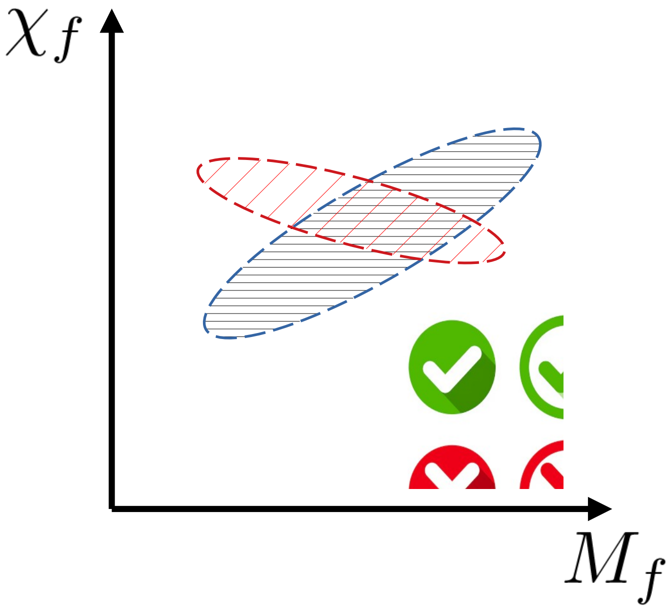

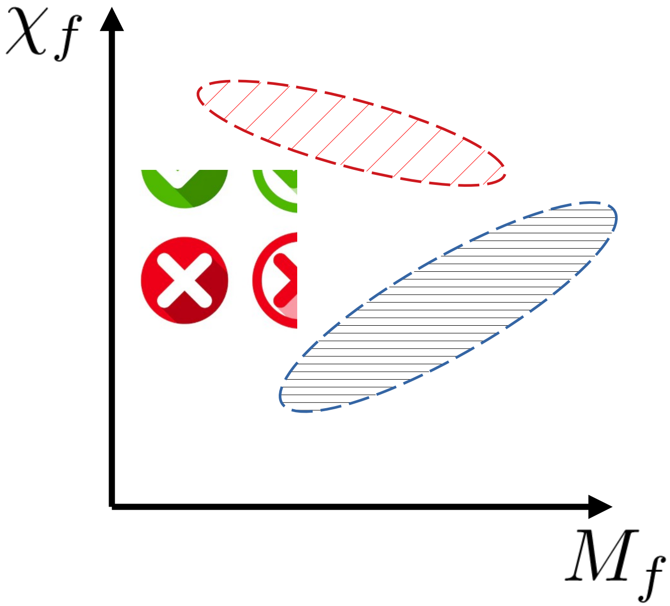

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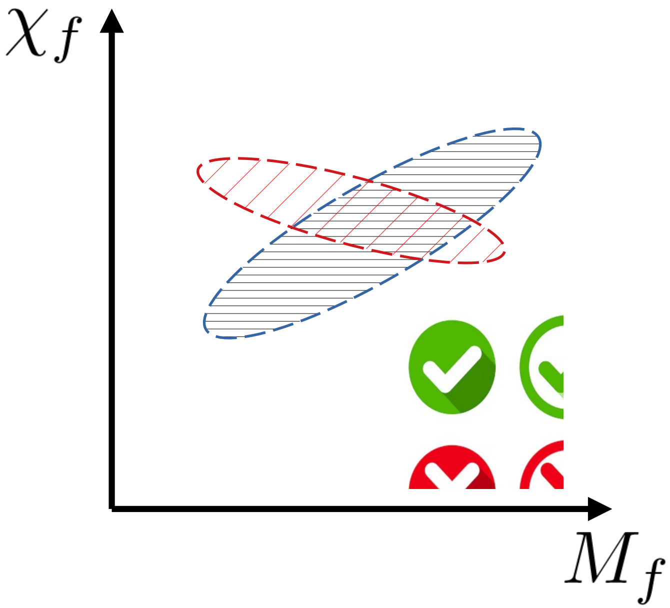
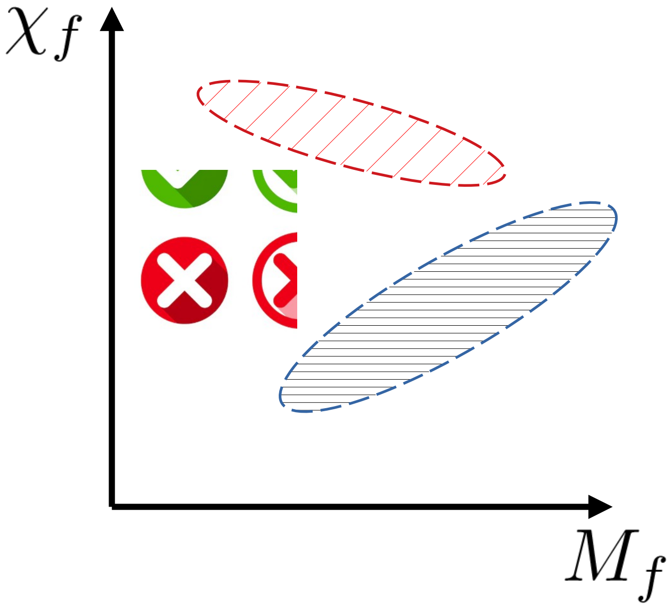
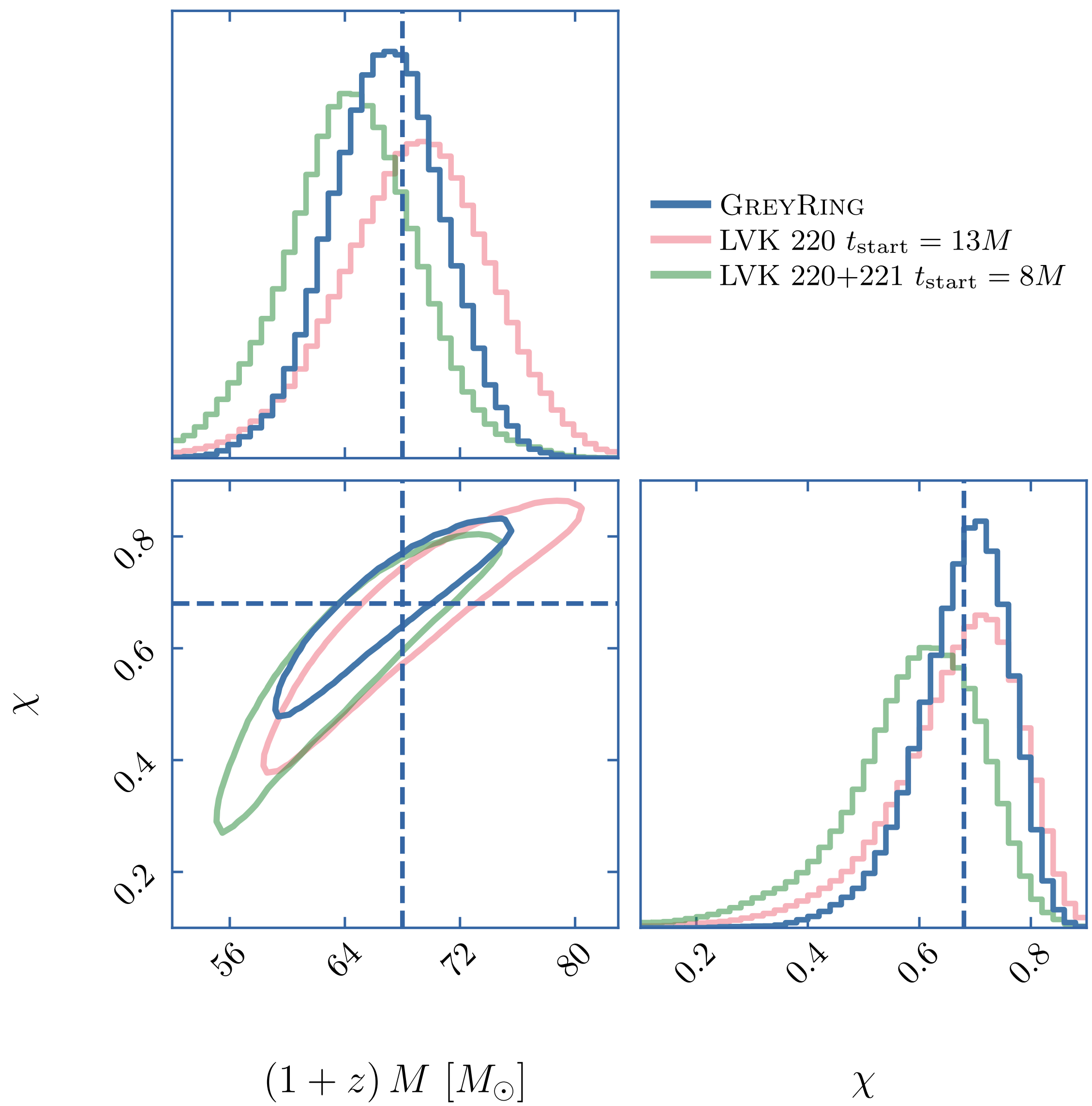


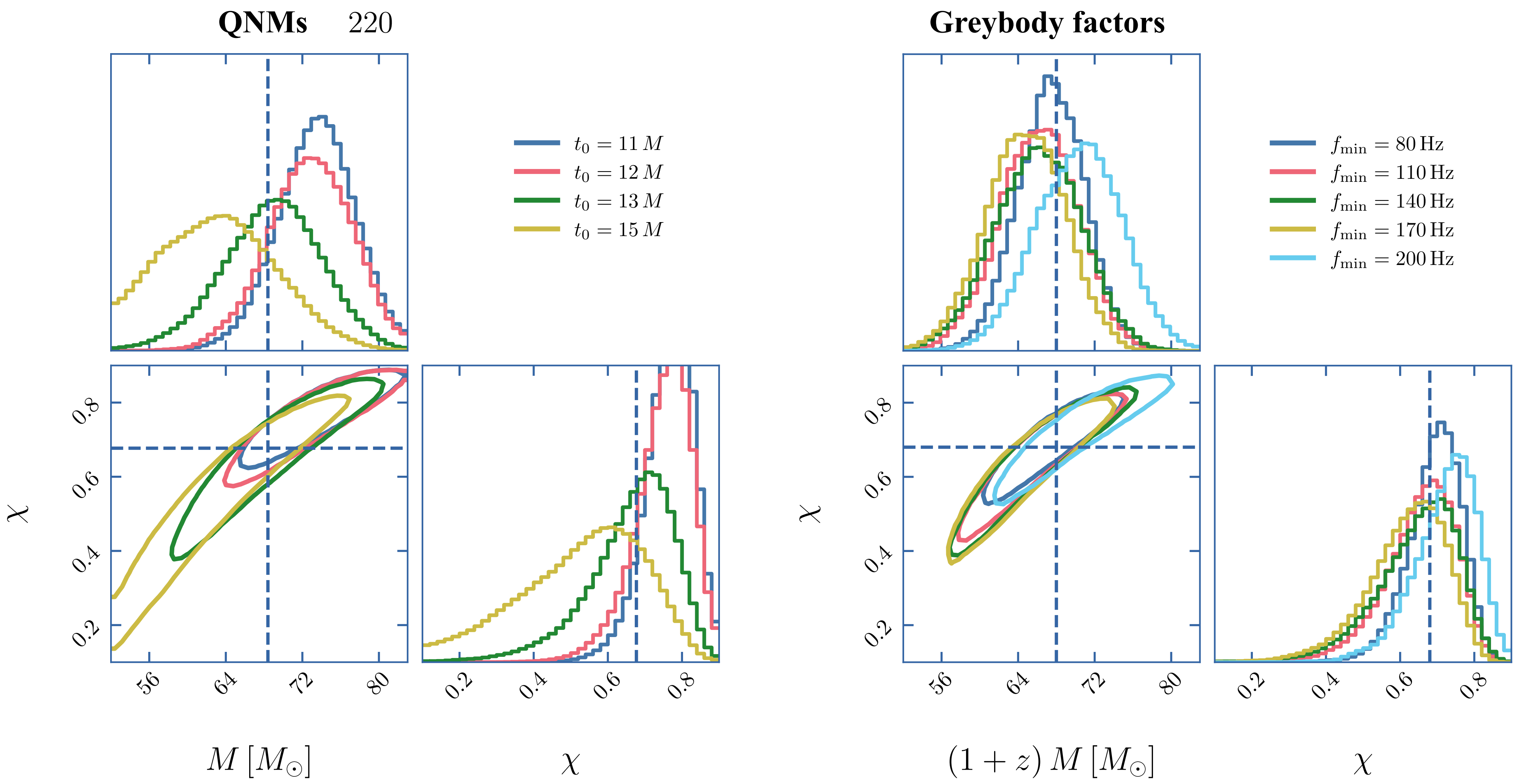


- Ringdown-only, agnostic consistency test
- Only dominant l=m=2 mode
- No overtones involved ("resummed")
- Entire frequency dependence

GW250114 with GreyRing



GW250114 with GreyRing

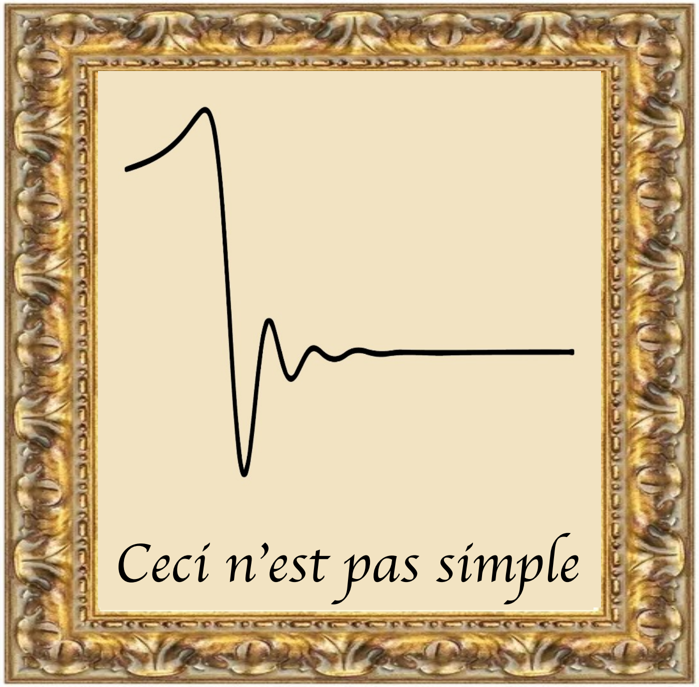


Dependence on starting frequency much weaker compared to dependence on starting time in ordinary spectroscopy

Conclusion & Outlook

Ringdown is complicated...but rich!

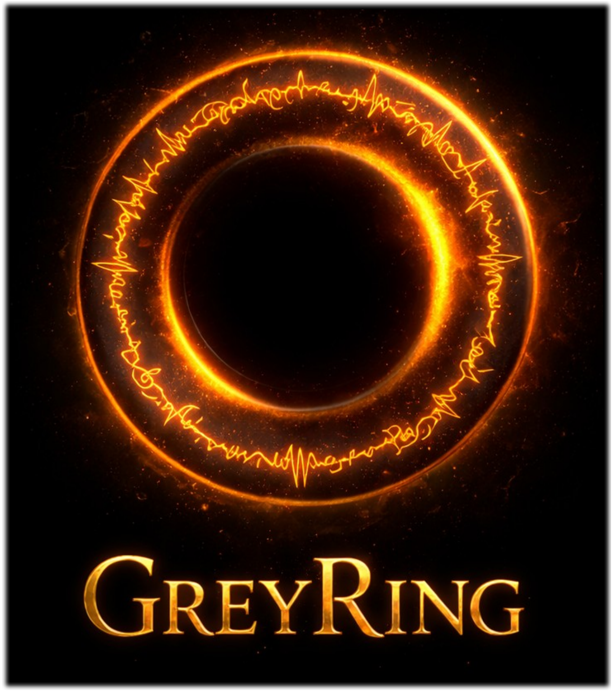
- QNMs (as eigenvalues/poles) do not necessarily describe time-domain signal
- Greybody factors imprinted in ringdown
- Novel tests of gravity



Work in progress with GreyRing :

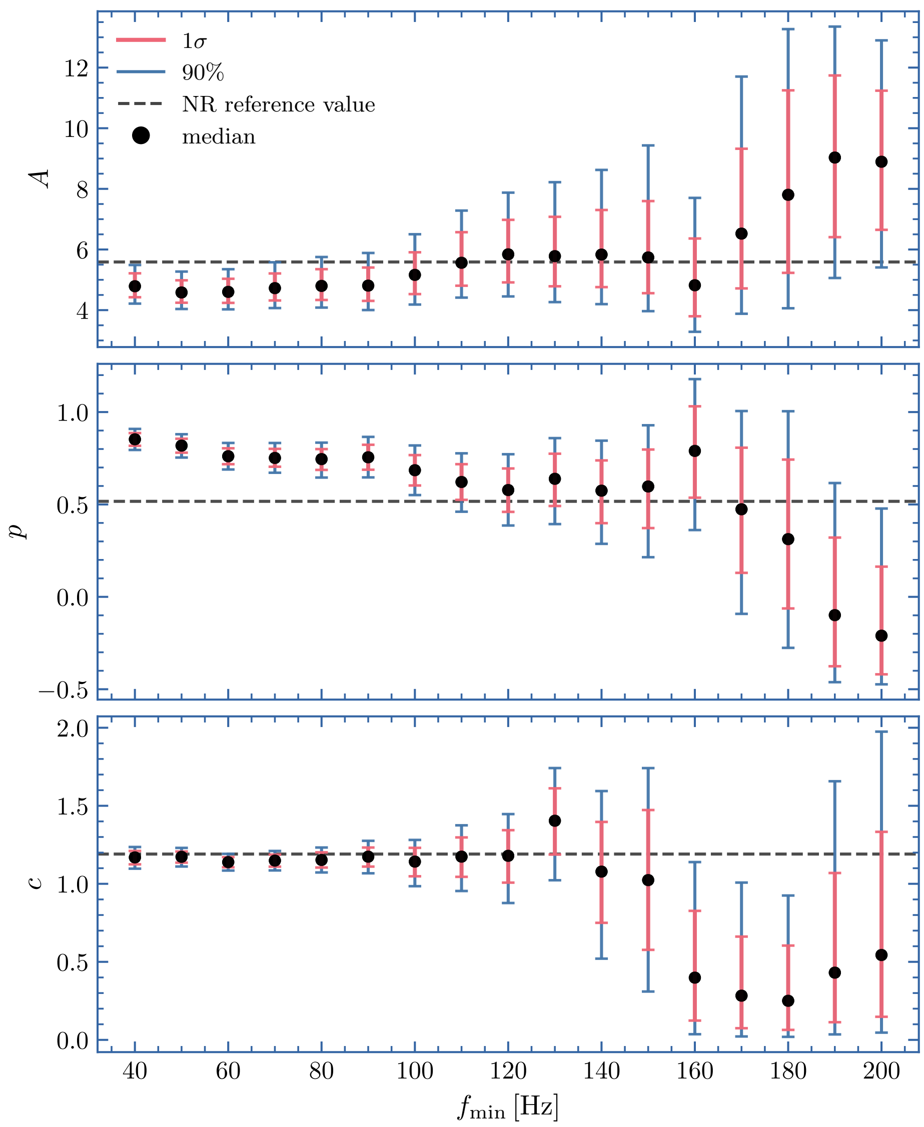
- Improved IMR waveform models with GFs
- Theory-specific tests
- Include mode-mixing, nonlinearities, ...
- Fitting formulas from NR, incl. precession

Thanks for your attention!



Backup slides

"Nothing is More Necessary than the Unnecessary" [cit.]

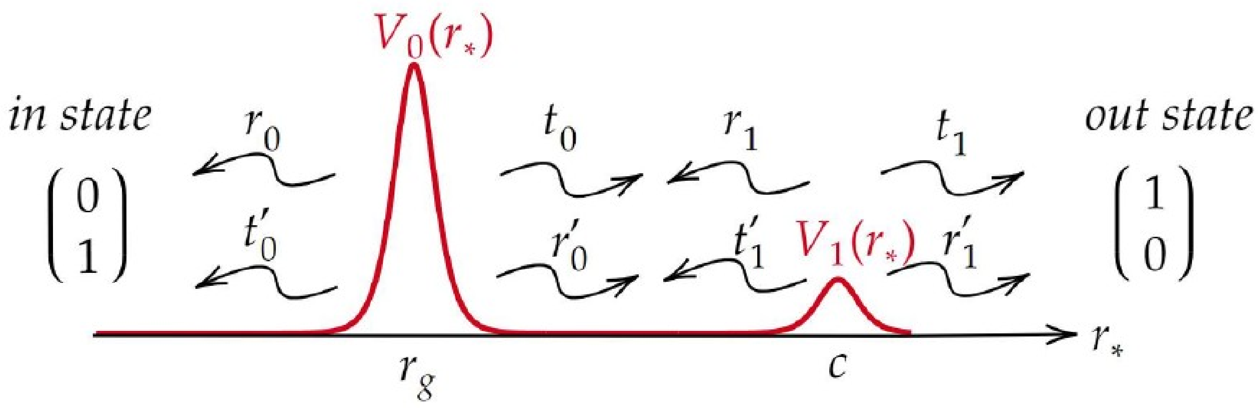


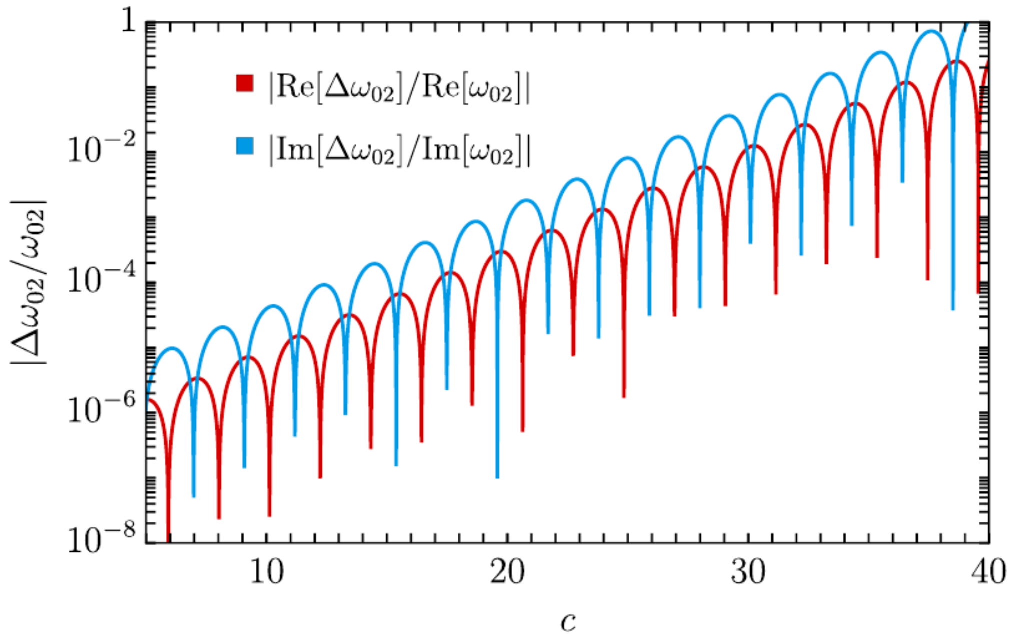
1σ
90%
NR reference value
median
A
p
c
$f_{\min}$ [Hz]

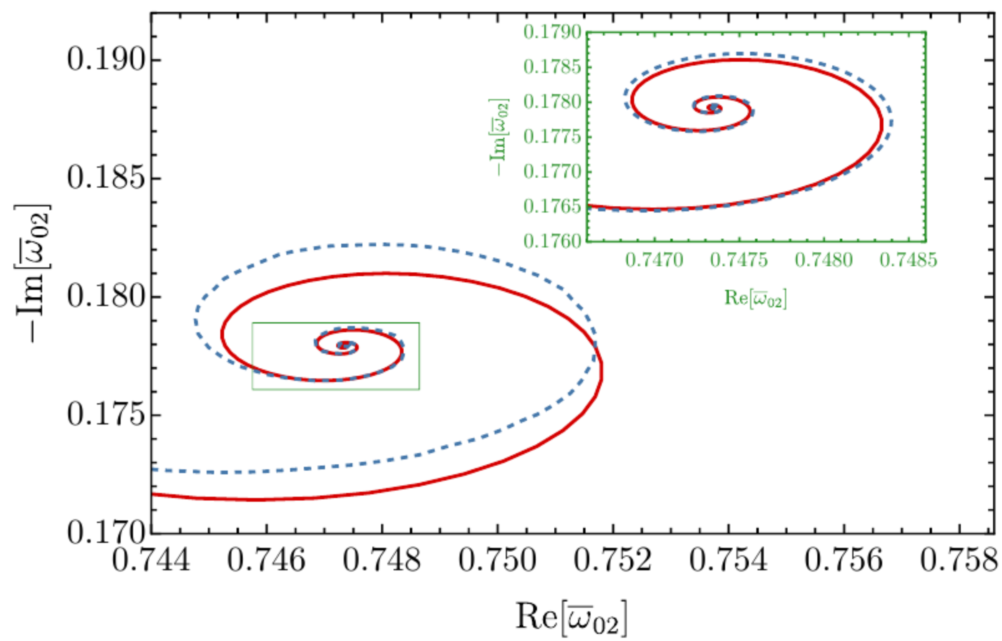
QNM spectral instability #3

Ianniccari+ PRL 2024

- Analytical treatment in terms of transfer matrix







Bonus Part

Black Hole Amplitudescopy

BH spectroscopy

- QNMs in GR depend only on mass and spin → Null-hypothesis tests
- Searches for GR deviations:

$$h(t) = \sum_i A_i \cos\left(2\pi f_i^{\rm Kerr}(1+\delta f_i)t + \phi_i\right) e^{-\frac{t}{\tau_i^{\rm Kerr}(1+\delta\tau_i)}}$$

$$f_i = f_i^{\rm Kerr}(1+\delta f_i) \qquad\qquad \tau_i = \tau_i^{\rm Kerr}(1+\delta\tau_i)$$

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Limitations of this approach:

1. $\delta f_i, \delta\tau = \text{const}$ → no theory with this property
2. $\delta f_i(\chi), \delta\tau(\chi)$ functions of coupling and spin → need to solve PDEs

Small-spin expansion

- High order needed
- Many parameters or theory dependent

ParSpec: Maselli, Pani+ 2020, 2024; Carullo 2021

Small-spin: Pani+ 2009, 2013; Molina+ 2010; Pierini-Gualtieri 2021, 2022; Wagle+ 2022, 2024; Cano+ 2022, 2023

Brute force numerical resolution

- Accurate, but theory dependent

Review: Dias, Godazgar, Santos

Applications: Chung 2023-2024; Blázquez-Salcedo 2024

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$$\tau_i^{\rm Kerr}(1+\delta\tau_i)$$

Can we find a ringdown test that is theory-agnostic, accurate, and practical?

Limitatio

1. $\delta f_i, \delta\tau = {\rm const}$ → no
2. $\delta f_i(\chi), \delta\tau(\chi)$ functions of coupling a to solve PDEs

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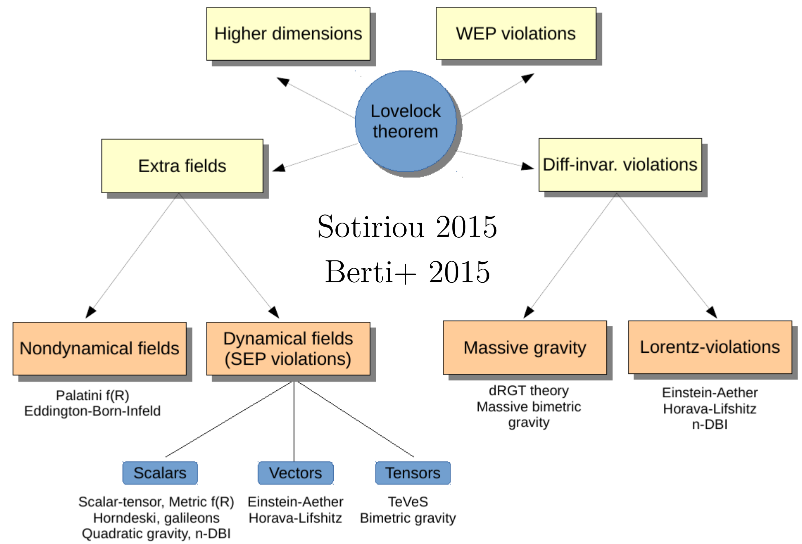
Not only QNM deviations

- **Lovelock theorem:** extra DOF are almost unavoidable beyond GR
  - **Scalar:** Horndeski, f(R)
  - **Vector:** Proca-Horndeski, Einstein-Aether, (Maxwell)
  - **Tensor:** massive/Weyl/quadratic gravity
  - EFT approach to GR
  - Low-energy string theory



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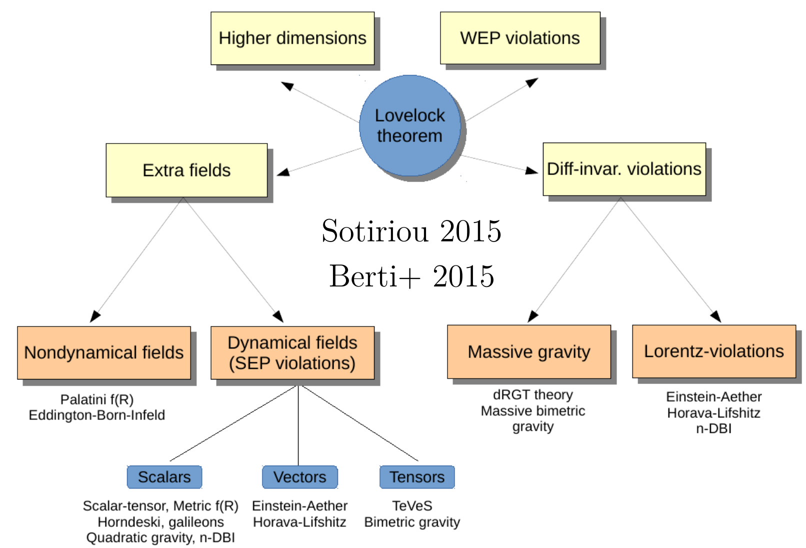

- Two generic predictions of extra fields in the ringdown:

✓ Deformation of the Kerr QNMs

✗ Extra modes in the gravitational signal, excited during the ringdown

This talk!

Example: Chern-Simons gravity

$$S = \frac{1}{16\pi}\int d^4x\sqrt{-g}R - \frac{1}{2}\int d^4x\sqrt{-g}g^{ab}\nabla_a\vartheta\nabla_b\vartheta + \frac{\alpha}{4}\int d^4x\sqrt{-g}\vartheta\,{}^*RR$$

Alexander-Yunes, 2009

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Nonspinning BHs = Schwarzschild, but different (axial) perturbations:

Cardoso-Gualtieri, PRD 2009

Molina+ PRD 2010

$$\frac{d^2}{dr_\star^2}\Psi+\left\{\omega^2-f\left[\frac{\ell(\ell+1)}{r^2}-\frac{6M}{r^3}\right]\right\}\Psi = \frac{96\pi\,Mf}{r^5}\alpha\Theta\,,$$

$$\frac{d^2}{dr_\star^2}\Theta+\left\{\omega^2-f\left[\frac{\ell(\ell+1)}{r^2}\left(1+\frac{576\pi M^2\alpha^2}{r^6\beta}\right)+\frac{2M}{r^3}\right]\right\}\Theta = f\frac{(\ell+2)!}{(\ell-2)!}\frac{6M\alpha}{r^5\beta}\Psi$$

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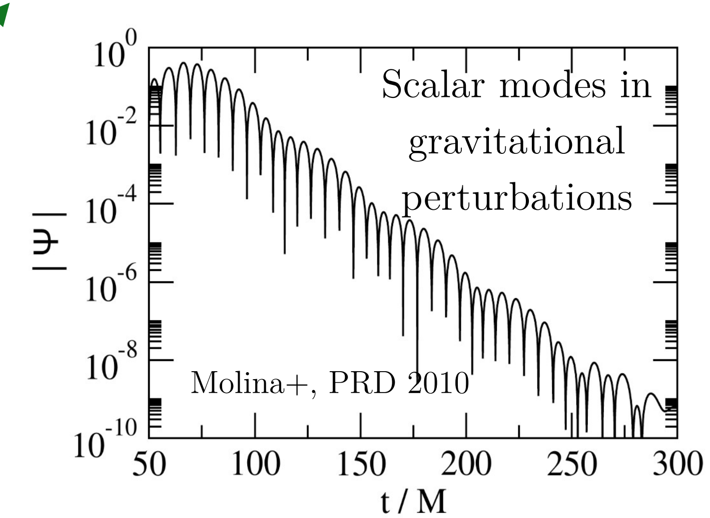
- Deformations in the effective potential
- Scalar & grav perturbations are coupled
    - Two classes of modes
    - Coupled system of harmonic oscillators



Unavoidable when extra fields couple to gravity:

- Gauss-Bonnet [Blázquez-Salcedo+ PRD 2016], Weyl gravity [Antoniou+ PRD 2025], even Maxwell

Modelling ringdown extra modes

Crescimbeni+ 2024-2025

Lestingi+ 2025

$$h(t) = \sum_i A_i \cos\left(2\pi f_i^{\rm Kerr}(1+\delta f_i)t + \phi_i\right) e^{-\frac{t}{\tau_i^{\rm Kerr}(1+\delta\tau_i)}}$$

$$+ \sum_i \hat{A}_i \cos\left(2\pi \hat{f}_i t + \hat{\phi}_i\right) e^{-t/\hat{\tau}_i}$$

$$\hat{f}_i = f_i^{{\rm Kerr},\, s=0}(1+\delta\hat{f}_i)$$

$$\hat{\tau}_i = \tau_i^{{\rm Kerr},\, s=0}(1+\delta\hat{\tau}_i)$$

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$$\hat{f}_i = f_i^{\rm Kerr,\, s=0}(1+\delta\hat{f}_i)$$

$$\hat{\tau}_i = \tau_i^{\rm Kerr,\, s=0}(1+\delta\hat{\tau}_i)$$

- **Crucial simplification:** amplitude proportional to the coupling → to leading order:

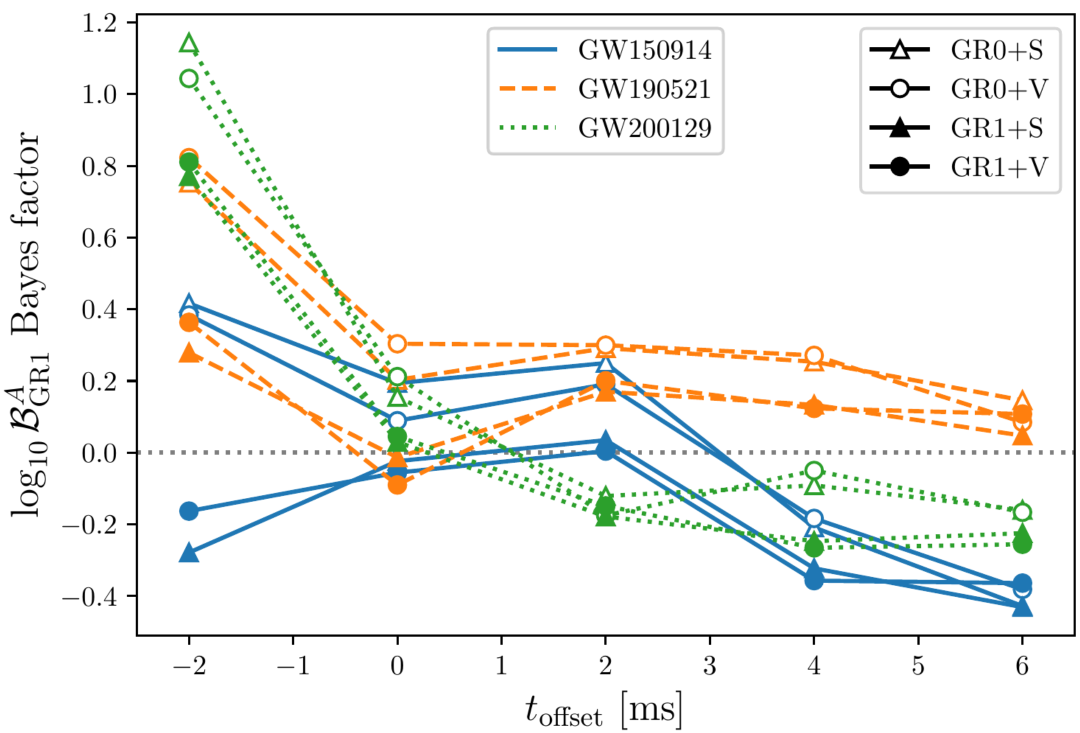
$$+ \sum_i \hat{A}_i \cos\left(2\pi f_i^{\rm Kerr,\, s=0} t + \hat{\phi}_i\right) e^{-\frac{t}{\tau_i^{\rm Kerr,\, s=0}}}$$

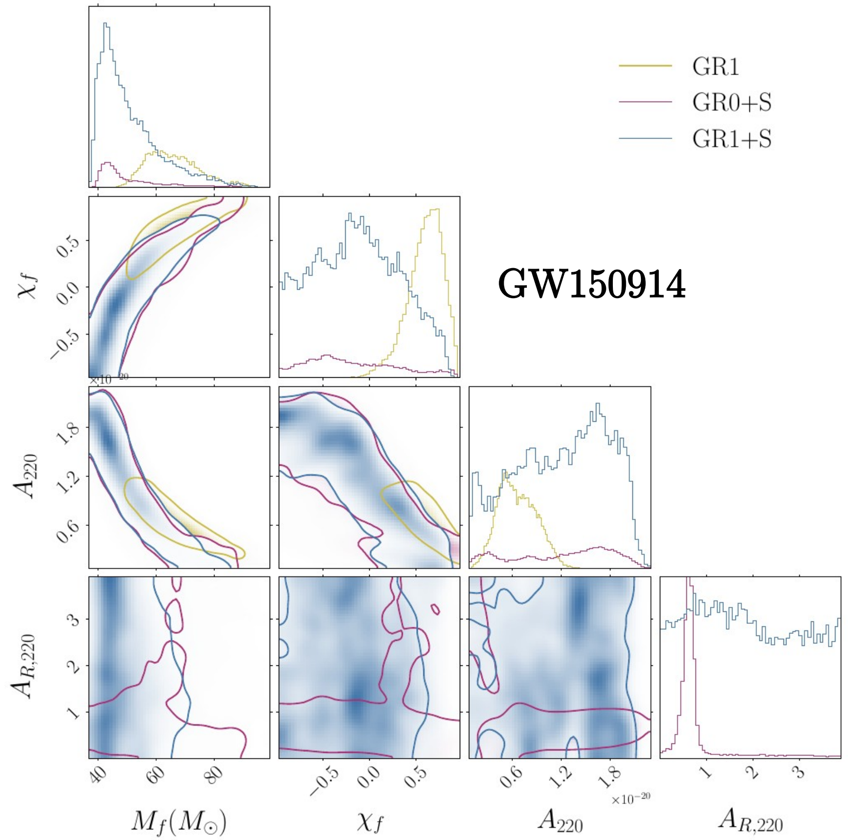
Pros:

- Kerr QNMs known for *any spin*, only beyond-GR parameters: **amplitudes** & **phases**!
- Same pattern function and spheroidal harmonics → Inclination angle factors out

Amplitudescopy on real data

- **GW150914**: golden event, still largest ringdown SNR, overtone debate [Isi+, Cotesta+ ...]
- **GW190521**: upper mass gap, tentative detection of l=3 mode [Capano+ 2021]
- **GW200129**: "false" GR deviations, tentatively ascribed to precession [Maggio+ 2022]





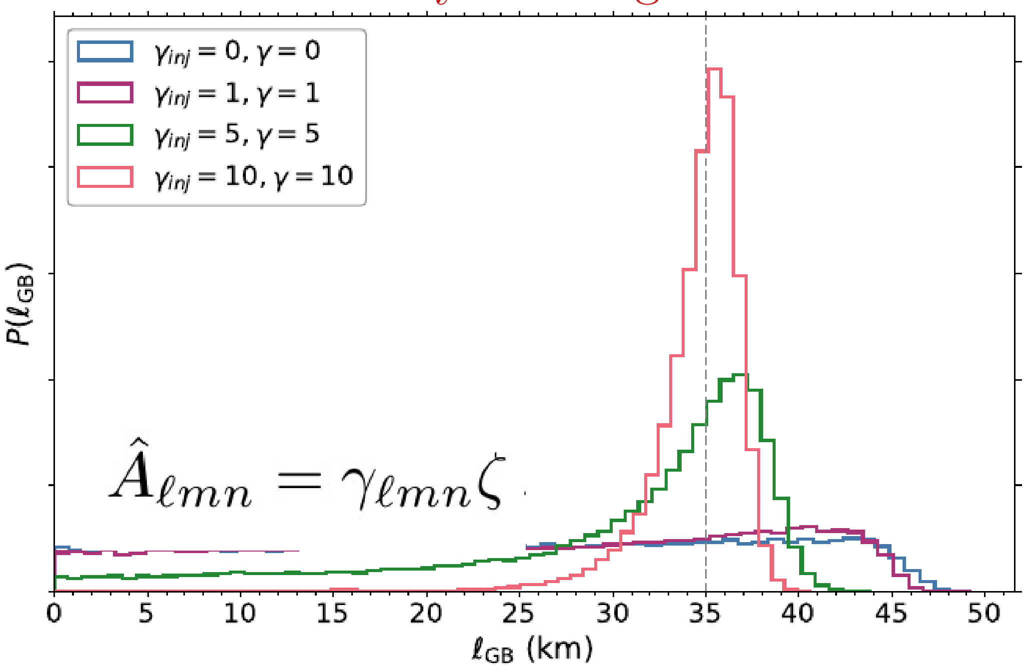
GR1 vs: GR0+S, GR1+S, GR0+V, GR1+V

→ no evidence

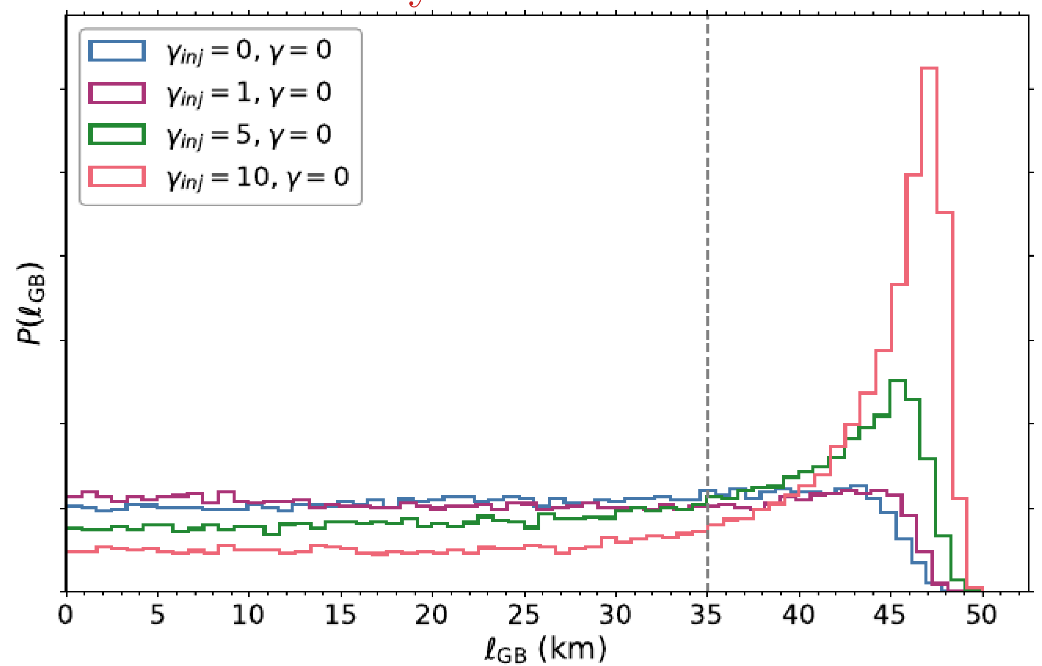
Scalar- or vector-driven modes affect parameter posteriors

QNM shifts + extra modes

Recovery with right model



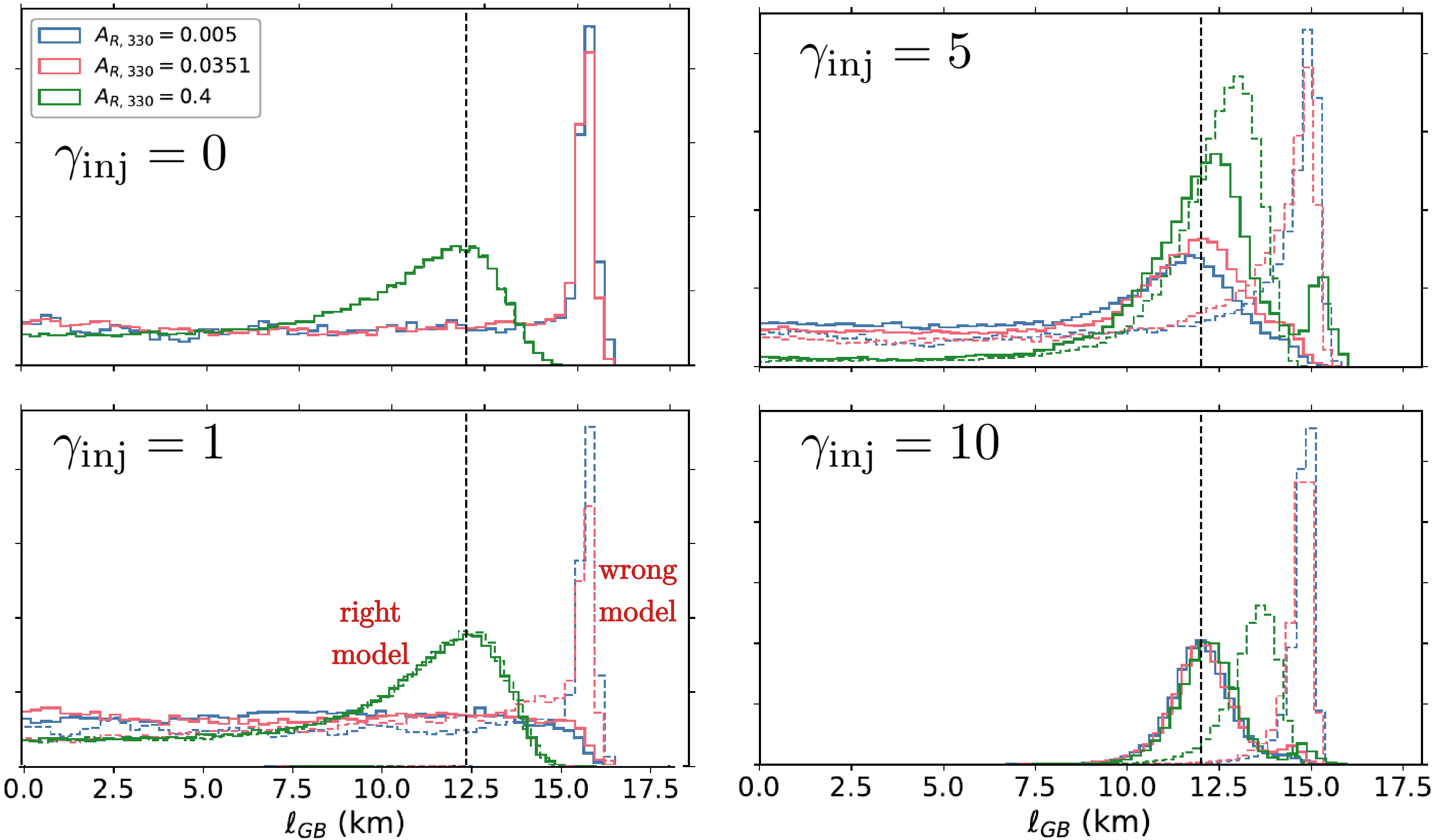
Recovery without extra mode



$$h(t) = \sum_i A_i \cos\left(2\pi f_i^{\rm Kerr}(1+\delta f_i)t + \phi_i\right) e^{-\frac{t}{\tau_i^{\rm Kerr}(1+\delta\tau_i)}}$$

$$+ \sum_i \hat{A}_i \cos\left(2\pi \hat{f}_i^{\rm Kerr,\, s=0} t + \hat{\phi}_i\right) e^{-t/\hat{\tau}_i^{\rm Kerr,\, s=0}} \,.$$

QNM shifts + extra modes

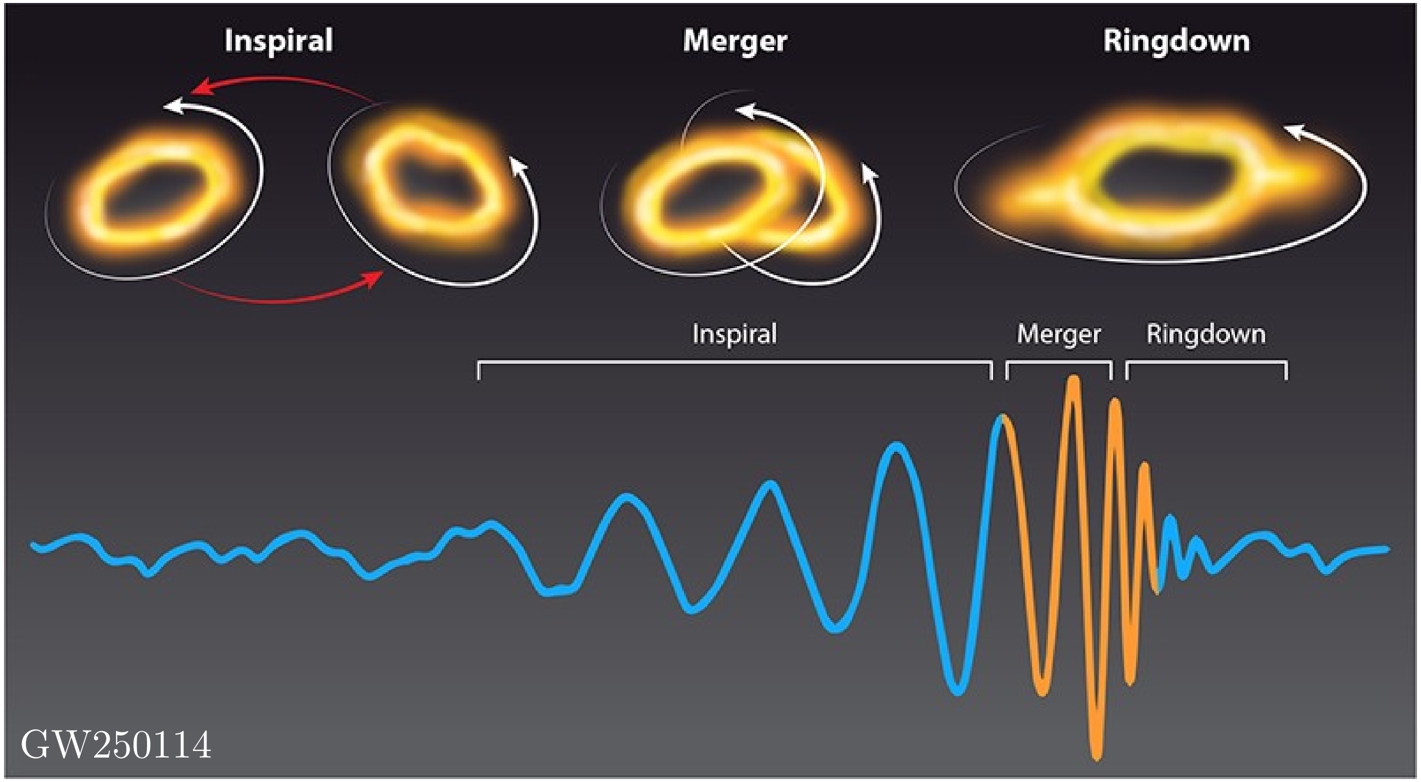


- Extra mode breaks degeneracy and allows using only fund. grav. QNM

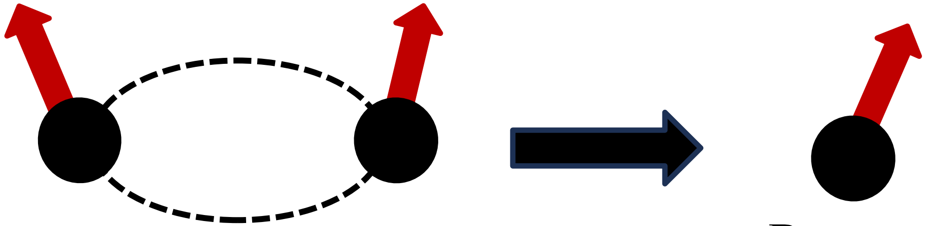
Part I

Amplitude-Phase Fits

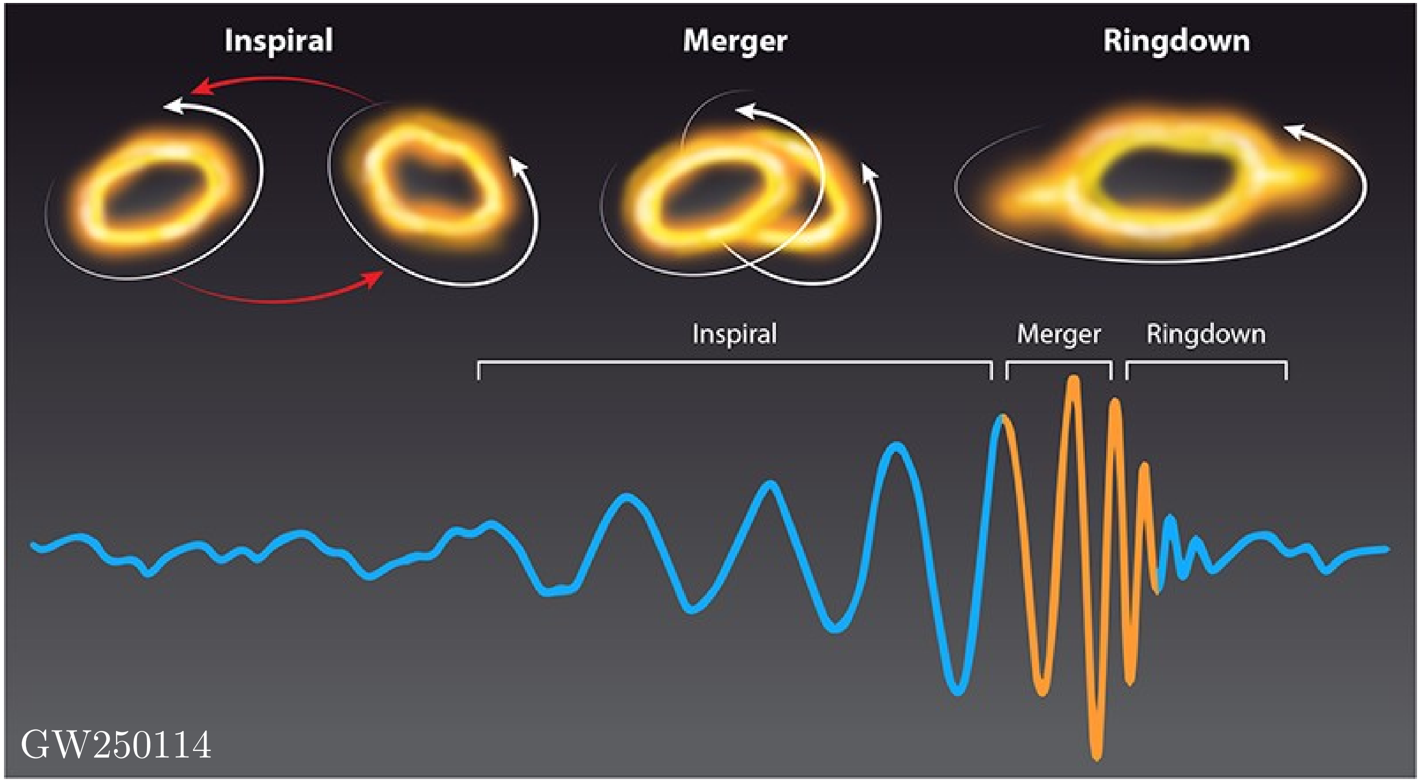
Ringdown



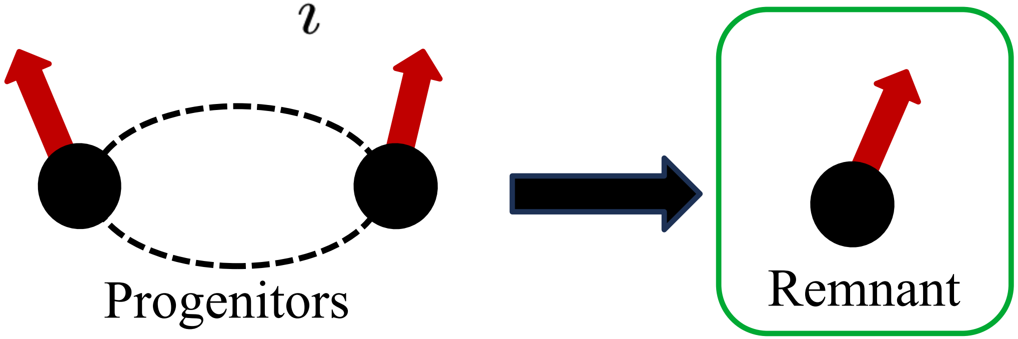
$$h_{\text{ringdown}} \sim \sum_i A_i \sin(\omega_i t + \phi_i) e^{-t/\tau_i}$$



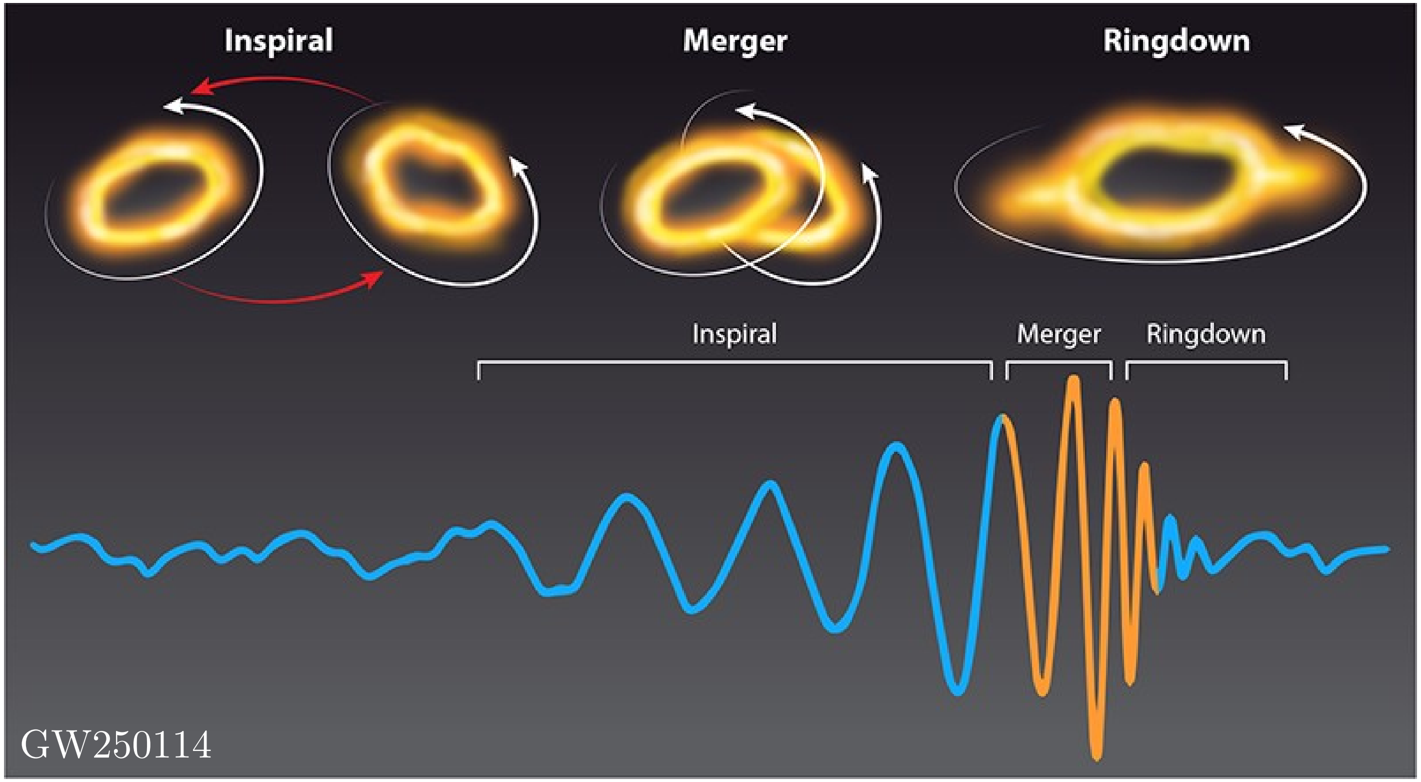
Ringdown



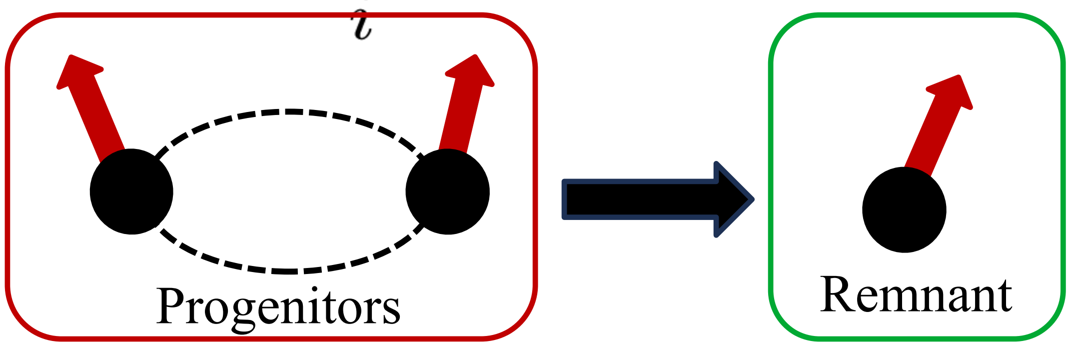
$$h_{\rm ringdown} \sim \sum_i A_i \sin(\omega_i t + \phi_i) e^{-t/\tau_i}$$



Ringdown



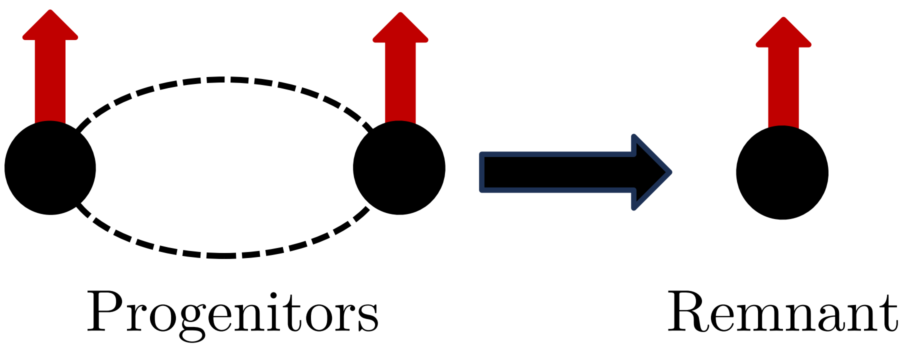
$$h_{\rm ringdown} \sim \sum_i A_i \sin(\omega_i t + \phi_i) e^{-t/\tau_i}$$



Modelling amplitudes & phases

Focus on circular, non-precessing binaries [see Anselmo+ 2512.05193 for precession]

- **London+**, 1810.03550, **Cheung+**, 2310.04489
- Forteza+, 2005.03260; Pacilio+, 2408.05276; Magaña-Zertuche+, 2502.03155, ...
- **TEOBPM** (Damour-Nagar, 1406.0401; Del Pozzo-Nagar, 1606.03952)
- SEOBNR (Y. Pan, A. Buonanno+, 1307.6232)

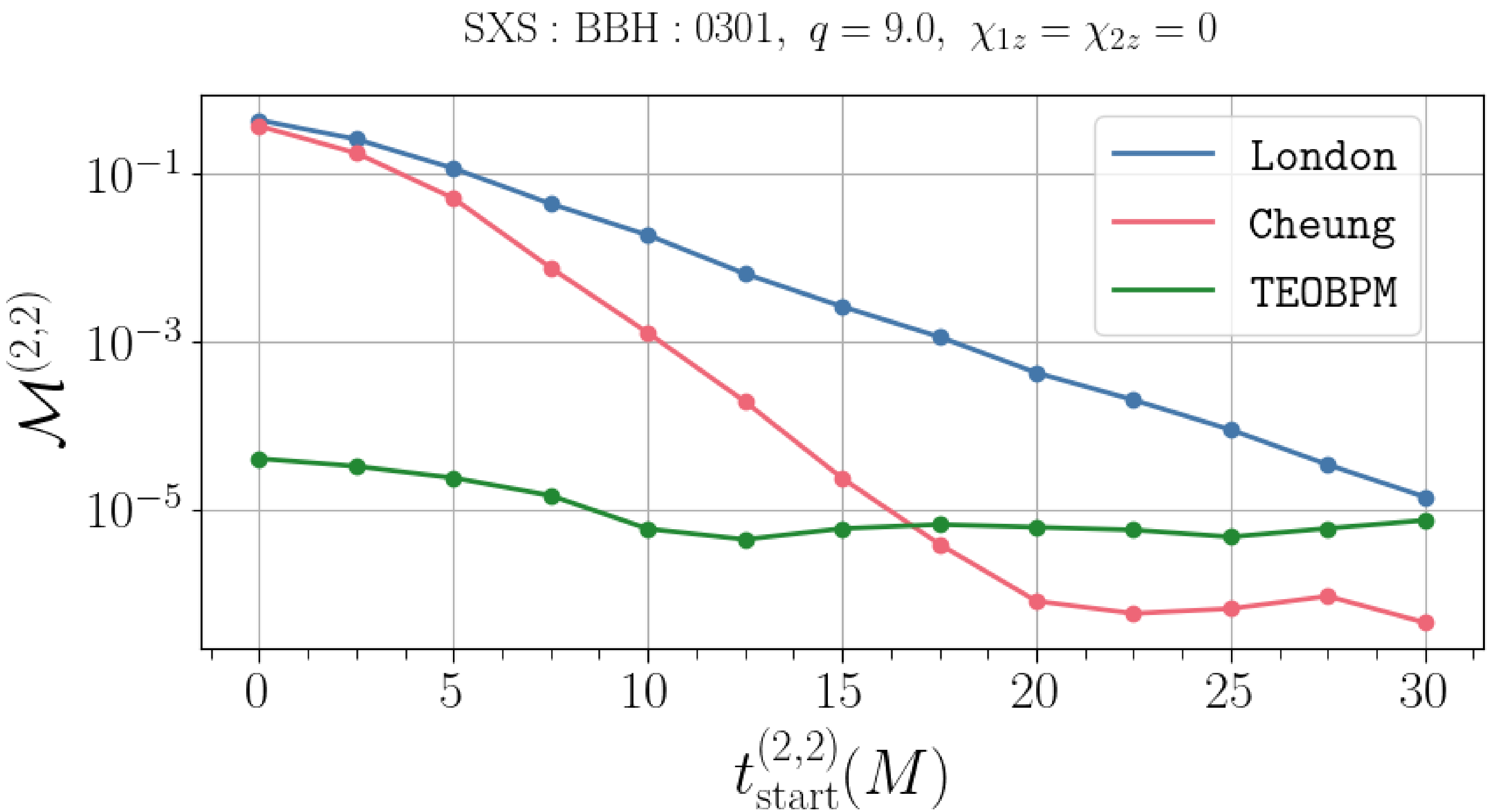


$$h_{\ell,-m} = (-1)^{\ell} h^{*}_{\ell,m}$$

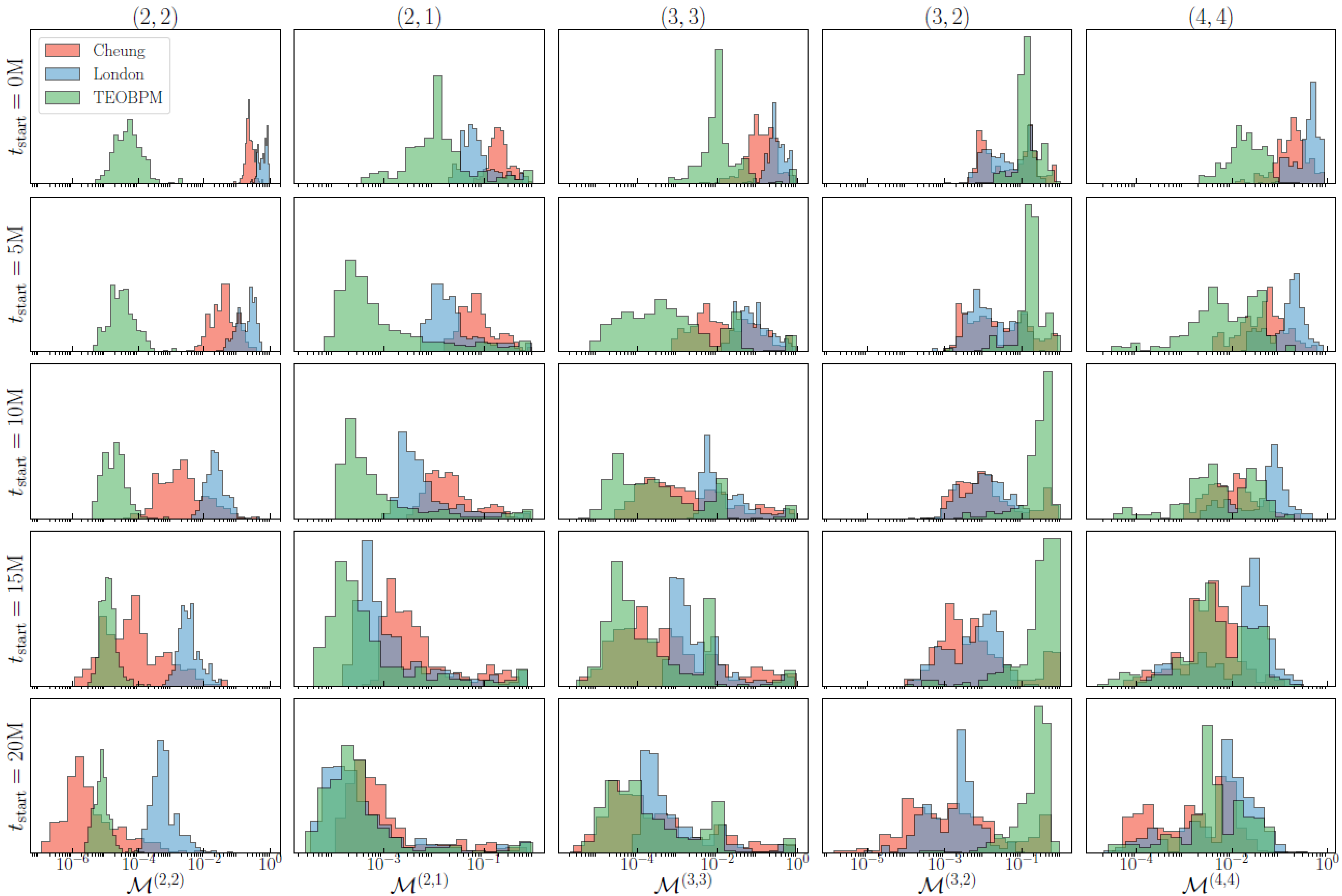
Fit model against Numerical Relativity simulations from SXS catalogue

Mismatch wrt NR

Crescimbeni+ 2511.02915

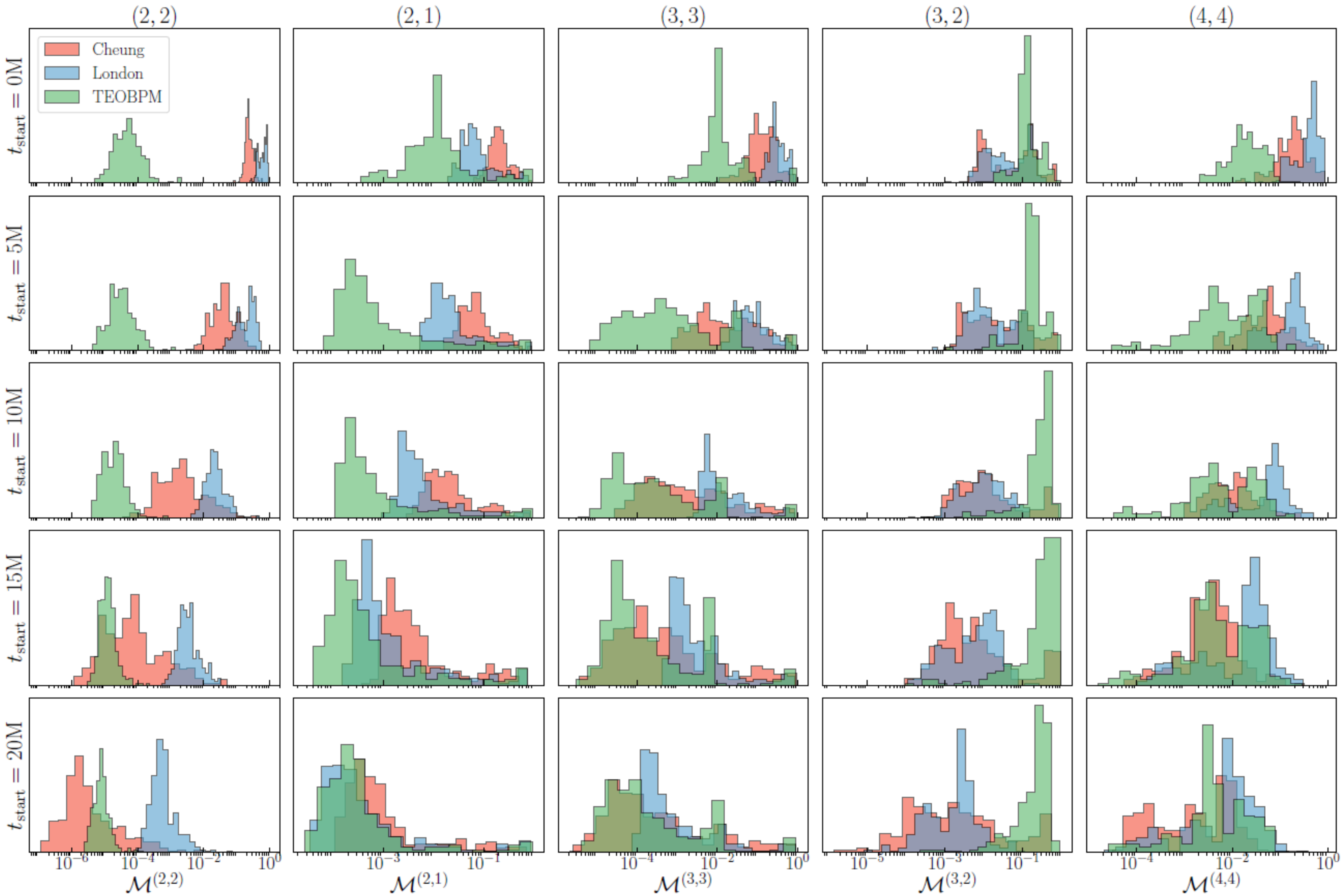


Mismatch wrt NR



Crescimbeni+ 2511.02915

Good or Bad News?



$$\mathcal{M} \lesssim \frac{1}{2\,\mathrm{SNR}^2}$$

Simplistic rule of thumb to avoid systematics

- ET/LISA will ringdown SNR~100-1000 [Bhagwat+ PRD 2022, Bhagwat+ PRD 2023]
- Are we ready for future detectors?

Amplitude-Phase consistency test